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Visibility of low-spatial-frequency sine-wave targets: Dependence on number of cycles

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The number of cycles in a low-frequency sinusoidal display is a crucial variable in determining the visibility of the display. In particular, the threshold contrast is essentially independent of spatial frequency for these displays. We have extended the above experiments, using more cycles and a variety of targets and observer tasks. The results confirm previous findings; they also show that the type of target or task has little influence. For low-frequency sinusoids that contain up to about 3 cycles, the threshold contrast is determined by the number of cycles. For high-number-of-cycles targets with spatial frequencies above 6–10 cycles per degree, visibility is predominantly dependent on the spatial frequency. The results suggest that the low-frequency decrease in reported MTF's is due to the decrease of the number of cycles used in determining them.

Index Headings: Vision; Modulation transfer.

In previous work,[1] we found that the number of cycles in a sinusoidal target can be crucial for determining contrast threshold. Our targets contained between $\frac{1}{2}$ and 3 cycles and had nominal spatial frequencies[2] between 0.1 and 3 cycles per degree. For these low-frequency, low-number-of-cycles targets, the number of cycles was sufficient to specify visibility. In particular, changing the spatial frequency did not affect visibility. Hoekstra, Van der Goot, Van den Brink, and Bilsen[3] obtained similar results. Numerous other investigators,[4–7] using a variety of procedures, have shown that spatial frequency is a critical variable for determining the visual system's sensitivity to high-frequency sine waves. So, for low-number-of-cycle, low-spatial-frequency targets, visibility depends on only the number of cycles and is independent of spatial frequency; for many-cycle, high-spatial-frequency targets, visibility depends on only spatial frequency. We undertook the present set of experiments to extend our examination to targets that have more than 3 cycles and spatial frequencies greater than 3 cycles per degree, to delineate the boundaries between the regions of influence of spatial frequency and number of cycles. We also wanted to test whether or not the particular targets and observer task that we used in our previous work played an important role in our results. We will briefly discuss the implications of the data with respect to linear-systems analysis of the visual system and the modulation transfer function (MTF).

EXPERIMENTS

We suspected that the important difference between our previous experiments and those of others was that we were measuring low-spatial-frequency, low-number of-cycle targets. However, there were other differences in the design of the experiments. To illustrate these differences, we will compare our procedure with some others in the literature. All of the targets to be discussed varied sinusoidally in one direction and were uniform in the perpendicular direction. The graph on the left in Fig. 1 is a luminance profile of one of our targets. The sine-wave portion of the target can be thought of as being added on top of a plateau of illumination. Thus, the sine wave is seen against a much darker background. The boundary of the sinusoidal portion is in the shape of a regular octagon. The observers' task was to detect the orientation of the sine wave. We made targets and task of this form in order that they would be compatible with others used in that set of experiments. The graph on the right hand side of Fig. 1 is a luminance profile of a typical stimulus used by DePalma and Lowry.[5] The boundary of the sinusoidal region was square. The background had the same average luminance as the center. In DePalma and Lowry's experiments, the subjects' task was to adjust the contrast of the target until it was just visible. This task will be referred to as threshold. Davidson[6] used targets that had a circular boundary against a dark background. In his experiments, the task was to select a target whose apparent contrast equalled that of a standard. This task will be referred to as contrast matching.

We decided to repeat and extend our previous examination of the influence of number of cycles on the visibility of sine waves, using the four procedures summarized in Fig. 2. There are two types of task (threshold and contrast matching) for each of two types of target (with and without plateau). The number of cycles in the displays ranged from $\frac{1}{2}$ to 80.

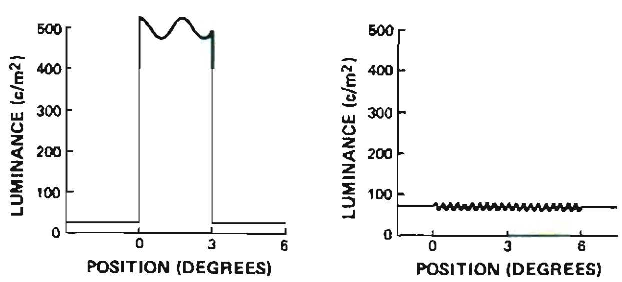

FIG. 1. Some of the differences between a typical target we used in a previous paper and a typical target used by DePalma and Lowry. In both cases, the targets were uniform in one direction and varied sinusoidally in the perpendicular direction. Our target, shown on the left, consisted of a small number of cycles at a low spatial frequency on a plateau. DePalma and Lowry's target, shown on the right, consisted of many cycles of sinusoid against a background of the same average luminance.

METHODS

Two types of targets were used. One had a uniform dark area around the sinusoidal center. The luminance in the octagonal, central region varied sinusoidally in one linear dimension and was constant in the orthogonal direction. The average luminance of the center was about ten times as much as the surround. These targets will be referred to as plateau targets. (See II and IV of Fig. 2.)

The second type of target had a different luminance in the surround. In these targets, the surround had the same luminance as the average of the square, central, sinusoidally varying region. The luminance of the surround was uniform and equal to 9.3 cd/m^2. These targets will be referred to as no-plateau targets. (See I and III of Fig. 2.)

Two types of judgment were used. In one case, called threshold, the subject could adjust the contrast of one display (the test target) or switch it to a uniform display. The switch was a make-before-break switch, which prevented transients from being visible to the observer. At contrasts less than threshold, the target and uniform display were indistinguishable; and there was no noticeable change when the switch was moved back and forth. The task was to adjust the contrast of the test target until it was just different from the uniform display; that is, just detectable. (See I and II of Fig. 2).

The second type of judgment consisted of adjusting the contrast of the test target until it was as visible as a $1\frac{1}{2}$ cycle, 10%-contrast standard target. This was a suprathreshold judgment, because the standard was clearly visible. As in the threshold judgment, the subject could switch the display from test to standard as well as control the contrast of the target. (See III and IV of Fig. 2.)

Figure 2 summarizes the four possible combinations of target and observer task. The four combinations are threshold with no plateau, threshold with plateau, contrast matching with no plateau, and contrast matching with plateau. For each combination, 10 targets were presented at each of three distances. The 10 targets consisted of $\frac{1}{2}$, 1, $1\frac{1}{2}$, 3, 5, 7, 10, 20, 40, and 80 cycles of sinusoidal oscillation. They were adjusted so that the maximum luminance occurred at the left edge of the sinusoidal region.[8,9] The three distances of 41, 97, and 290 cm resulted in displays whose sinusoidally varying portion subtended 7.6, 2.7, and 0.83 degrees of visual angle, respectively.

Two subjects viewed the targets monocularly at eye level. They each had normal visual acuity when they used corrective lenses. Each subject made 10 consecutive contrast settings. The average of these 10 settings determined the data points. The contrast of a sine-wave target is defined as the difference between the maximum and minimum luminance divided by the sum of the maximum and minimum luminance. Contrast sensitivity is the reciprocal of contrast.

The sinusoidal portion of the displays was generated on the cathode-ray tube of a Tektronix 535 oscilloscope, by use of techniques described elsewhere.[1] The 13-cm-diam surround was generated by a box mounted on the oscilloscope perpendicular to the tube face. The box contained twelve 7-W incandescent lamps controlled by a dimmer and enclosed by a baffle such that no lamps were visible to the subject. The light from the lamps was reflected by the white walls onto the matte white back of the box, which was seen as the uniformly illuminated surround for the target. For the no-plateau experiments, we placed a mirror surface (Alzak aluminum) at 45° to both the oscilloscope face and the uniformly illuminated surface. A hole was cut in the aluminum, such that the hole appeared square to the observer. The uniform surround was reflected by that mirror; the sinusoidal pattern was transmitted through the hole in the mirror (see upper diagram in Fig. 3). For the plateau experiments, we substituted a semi-silvered glass mirror for the aluminum. We placed a dark-gray paper mask on the semi-silvered mirror. The hole in the mask appeared to be a regular octagon when viewed at 45°. The plateau targets were the sum of the reflected uniform illumination and the transmitted

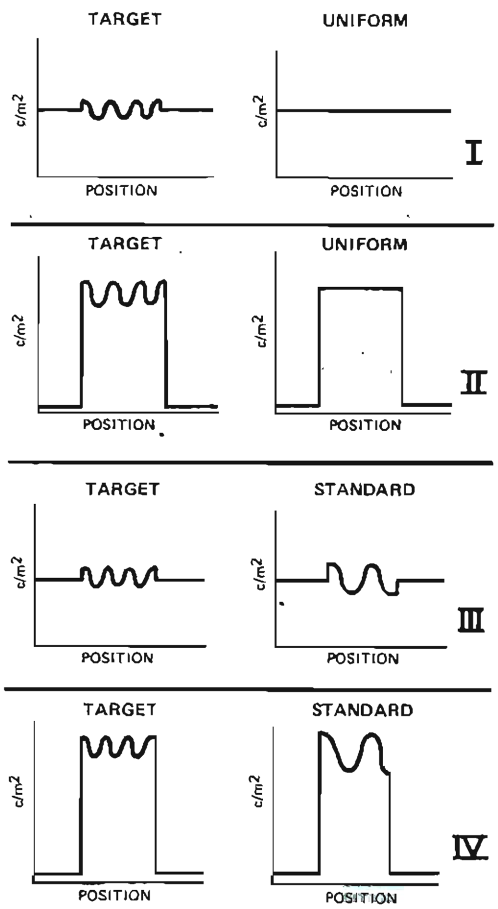

FIG. 2. Procedures used in the experiments. In I and II, the subjects adjust the contrast of the target until it is just distinguishable from the uniform display. In III and IV, the subjects adjust the contrast of the target until it is as visible as a 0.1 contrast, $1\frac{1}{2}$-cycle standard. The procedures I, II, III, and IV are referred to as threshold with no plateau, threshold with plateau, contrast matching with no plateau, and contrast matching with plateau, respectively.

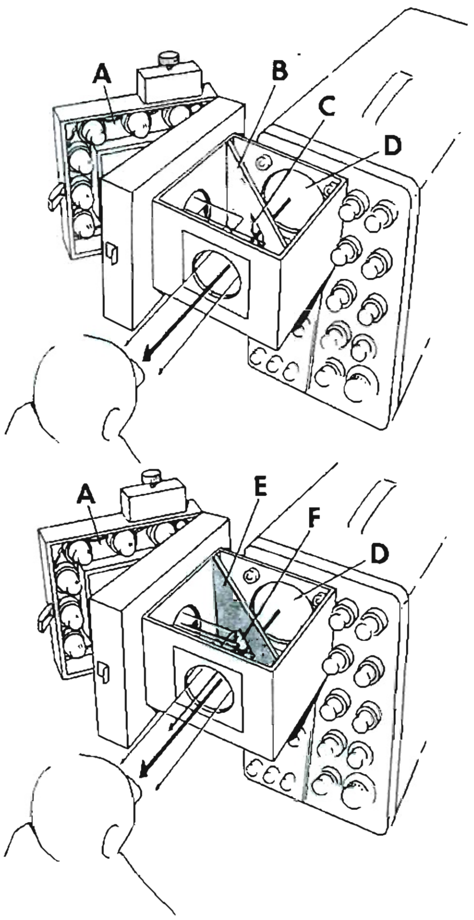

FIG. 3. Apparatus used to generate the stimuli. The no-plateau targets were generated by use of the upper arrangement, where A, B, C, and D designate the source of uniform illumination, mirror for average background, rectangular hole, and source of sinusoidal illumination, respectively. The with-plateau targets were generated by use of the lower arrangement, where A and D are as above and E and F designate the dark-grey background and octagonally shaped, partially silvered mirror, respectively.

sinusoidal pattern from the oscilloscope. A colored filter was placed between the subject and the mirror so that the entire display appeared to be of one color (green). This was necessary because the uniform illumination was white, whereas the tube signal was light blue.

The raw data consisted of voltage measurements of the input to the Z axis of the cathode-ray tube. This voltage was translated into contrast by use of graphs generated by use of a Gamma-Scientific scanning telephotometer to measure the contrast of the sine-wave gratings as a function of the Z-axis voltage. This was done for the $\frac{1}{2}$, 1, $1\frac{1}{2}$, and 5 cycle targets. The reason why several calibration graphs were needed is that the oscilloscope uses a capacitor to decouple the dc component of the Z-axis input. This capacitor noticeably attenuated the contrast for a given input voltage at the low frequencies (200–700 Hz) used to generate the targets with a small number of cycles. Calibration experiments showed that there was little change of the relation between voltage and contrast for targets with more than 5 cycles, so the graph for the 5-cycle target was used for higher numbers of cycles. Correction for the nonlinearity of the cathode-ray-tube luminance as a function of voltage was unnecessary, because we were operating over a small range of luminances, far from zero luminance.

RESULTS

For a fixed display size, each target can be specified either by the spatial frequency of the sine wave or by the number of sinusoidal oscillations. The data are plotted twice: once as contrast sensitivity versus number of cycles and once as contrast sensitivity versus spatial frequency (see Figs. 4 and 5). The data are for observer RLS; observer JAH gave similar results.

We immediately notice the similarity of the results for the different experiments. For a low number of cycles (up to 3 cycles), the visibility of a target that has a fixed number of cycles is approximately constant, despite an almost tenfold change of spatial frequency. This constant visibility holds even when the spatial frequency of the target is as much as 3.6 cycles per degree. (The 3-cycle target viewed from the farthest distance tested had this spatial frequency.) At spatial frequencies above about 5 cycles per degree, the curves start to converge when plotted on the spatial-frequency axis. Perfect convergence would suggest that spatial frequency is the determinant of visibility for many-cycle targets at high frequencies.

These results suggest two major conclusions. First, they confirm the dominant role that number of cycles plays in the visibility of low-number-of-cycles targets. Second, the results show that the luminance of the surround, the shape of the sinusoidal area, and the observer task can all be changed without changing the importance of number of cycles.

Having pointed out the general similarity of the results from the four different experiments, let us discuss the differences. The contrast-matching experiments are suprathreshold. That is, the subjects match the contrast of a test target to the contrast of a clearly visible standard. Consequently, the contrast sensitivities are lower for the matching procedure than for threshold. The contrast sensitivity is highest in the threshold-with-no-plateau case. The threshold-with-plateau case yields 50% lower contrast sensitivities than the threshold-with-no-plateau case. The addition of a plateau causes roughly the same decrease of threshold sensitivity, independent of the spatial frequency of the sinusoid.

The peak sensitivities of the curves in all four cases for targets that subtend 0.83° are never as great as those for targets that subtend 2.7° and 7.6°. We believe that this illustrates the combined influence of number of cycles and spatial frequency. As the number of cycles increases, the sensitivity for a given spatial-frequency target increases. As the spatial frequency of a target increases, the sensitivity of the visual system decreases

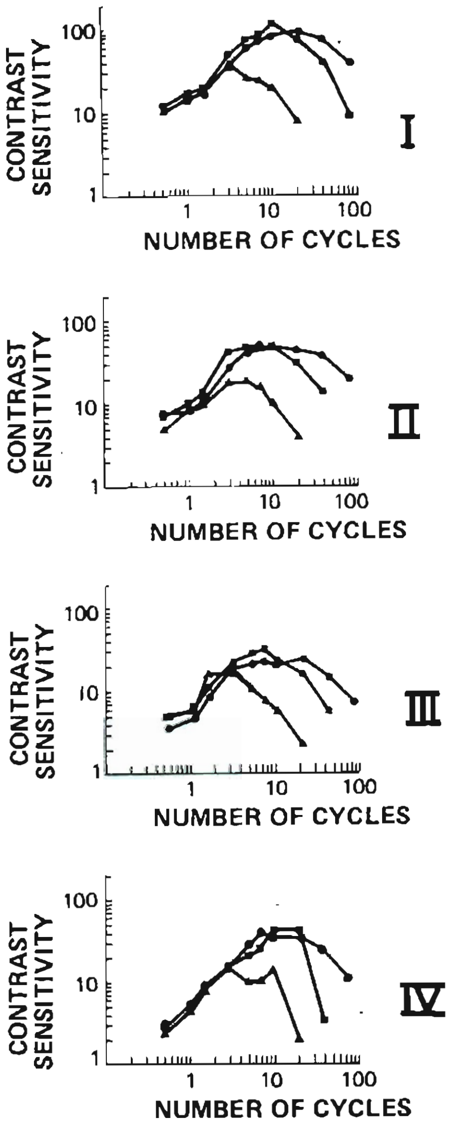

FIG. 4. Data for subject RLS in each of the four experiments. The contrast sensitivity is plotted against the number of cycles in the stimulus. The numbers I, II, III, and IV refer to threshold with no plateau, threshold with plateau, contrast matching with no plateau, and contrast matching with plateau, respectively. These are the same designations as are used in Fig. 2. Filled circles, squares, and triangles refer to targets subtending 7.6°, 2.7°, and 0.83°, respectively. For stimuli with a small, fixed number of cycles, but different sizes, and hence different spatial frequencies, the contrast sensitivities depend on the number of cycles.

(at least for spatial frequencies greater than about 3 cycles per degree[7]). In the case of the targets that subtend 0.83°, when 5 cycles are present, the spatial frequency is 6 cycles per degree. So, the increasing visibility with increasing number of cycles is offset by the decreasing visibility with higher spatial frequencies before the peak sensitivity is reached. In the Appendix, the qualitative and quantitative aspects of our data are summarized by use of relatively simple equations. The equations are based on the principal finding illustrated by the data—namely, that the number of cycles is the dominant influence in one region, that the spatial frequency is dominant in another region, and that the combined influence of both properties of the stimuli determines the contrast sensitivity in the intermediate region.

DISCUSSION

Role of number of cycles in other work

Display size has been discussed previously in the literature and recognized as having an influence on threshold determinations. Campbell and Robson[7] studied the influence of display size in a control experiment designed to test whether they could use different display sizes to get a greater range of spatial frequencies with their equipment. They found that for suffi-

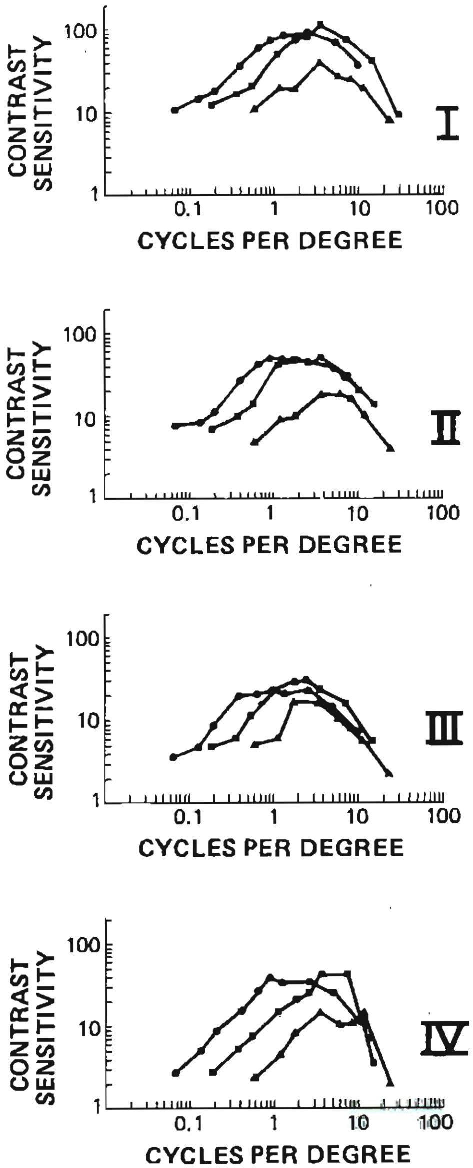

FIG. 5. Same data as Fig. 4, but here the horizontal axis is the spatial frequency of the stimuli. Otherwise, the notation is the same as in Fig. 4. The principal observation to be made is that the curves for stimuli of various sizes start to converge at frequencies greater than 5 cycles per degree.

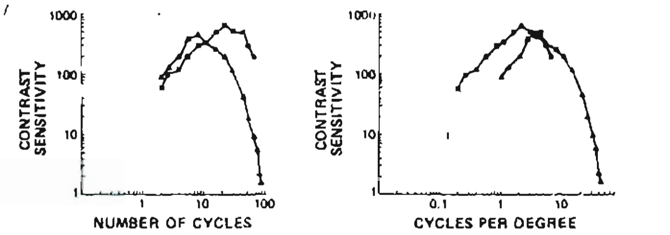

FIG. 6. Data of Campbell and Robson. Filled squares and triangles refer to targets subtending 10° and 2°, respectively. They plotted the contrast sensitivity versus spatial frequency, as shown on the right. We have replotted their data as a function of number of cycles on the left to show the near coincidence of the two curves in the low-number-of-cycles region.

ciently high frequencies, the display size had no effect. On the other hand, they noted that with less than about 4 cycles, thresholds were raised significantly. Part of their control experiment was similar to our experiment in that it consisted of measuring contrast sensitivity for the same display at two different distances. We have plotted Campbell and Robson's data as they did and then replotted it on the number-of-cycles axis to illustrate the overlap in the low-number-of-cycles region (see Fig. 6). This coincidence is what our results would suggest.

DePalma and Lowry,[5] in studying the influence of distance itself on sine-wave threshold, used larger targets at farther distances so that the visual angle subtended remained constant. However, for the farthest distance, they used a smaller target. This meant that for their farthest distance, they were performing the same experiment that we did. We replotted DePalma and Lowry's data,[5] as well as the data of Campbell and Robson.[7] Only the curve for the farthest distance shifted relative to the others when it was thus replotted. This shift had the effect of making the low-number-of-cycles portion of all of the curves overlap, as we expect from our data.

Hoekstra, Van der Goot, Van den Brink, and Bilsen[3] independently discovered the dominant role played by number of cycles for some stimuli. In addition, they varied the average luminance and found that with greater luminance the domain of influence of number of cycles was greater. For example, at a luminance about 60 times higher than ours, they found that the number of cycles determined threshold up to about 8 cycles. With luminances near ours they found, as we did, that the dominance of number of cycles ended near 5 cycles. Hoekstra *et al.* pointed out that the lowest spatial frequency they used was 0.5 cycles per degree. They suggested that it would be interesting to see if the importance of number of cycles continues at lower spatial frequencies. Our paper answers that question in the affirmative.

Fourier analysis and the MTF

Let us begin by examining, in both the spatial and spatial-frequency domains, the relationship between the three variables of interest in these experiments: width of the display (w), nominal spatial frequency of the display (f), and the number of cycles in the display (n).

Any two of these three variables determine the third, because they are related by the equation $w \cdot f = n$. How is this relation manifested in the spatial-frequency domain? Perhaps the clearest way to answer this question is to see what happens to the transform as we keep one of the three variables fixed and vary the other two, as shown in Fig. 7. The conclusion to be drawn is that the number of cycles does not determine any immediately apparent property of the transform, such as central frequency (which is determined by f), or bandwidth and peak value (which are determined by w, as long as n is not very close to zero[10]). Because f and w are related by $w \cdot f = n$, the properties of the transform that correspond to w and f will be related in the same way to the properties of the transform that correspond to n. For a fixed number of cycles, the product of peak value and central frequency (and the ratio of bandwidth and central frequency) is fixed.

There is extensive literature discussing the usefulness and validity of applying linear-systems analysis to the visual system. The with-plateau experiments of the present paper have discontinuous edges and such edges present serious problems for the linear-systems approach. Other papers[1,11] discuss this issue in more detail.

However, the principal point to be made with respect to our data and the linear-systems approach is that we have measured the threshold-contrast-sensitivity curve for each of three different display sizes and have found a different curve for each size. The question is whether we can consider the visual system to possess a single modulation transfer function (MTF), or must we associate a different MTF with each display size? If, as is

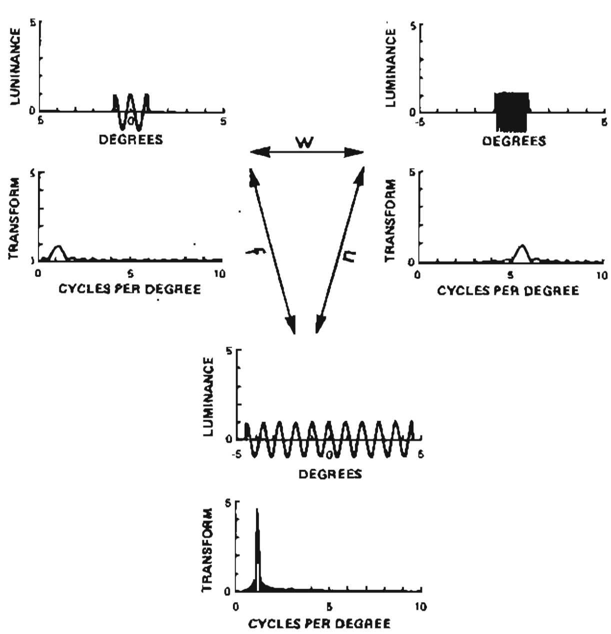

FIG. 7. Luminance profiles and magnitude of the Fourier transforms of three targets similar to those used in the experiments. The luminance profiles are directly above the transforms. The arrows point to the luminance profiles of targets with equal widths (n), equal nominal spatial frequencies (f), or an equal number of cycles (n).

frequently done in the literature, the threshold-contrast-sensitivity curve as measured under a fixed set of conditions is taken to be the MTF, then a different MTF would be required for each display size. We attempted to derive a single curve, which, combined with hypothetical spatial-frequency channels, describes the visual system's relative sensitivity to sine waves. It should be noted that this curve is a measure of the relative sensitivity of spatial-frequency channels and is not the same as the MTF for a linear system. Using assumptions and calculations described in detail in the Appendix, we derived a curve consistent with the data for all the no-plateau displays considered in this paper. The assumptions include spatial-frequency channels with bandwidths that depend on the frequency to which the channel is tuned. The spatial-domain interpretation of such channels involves receptive-field organizations with a fixed number of cycles; this implies larger receptive fields for channels tuned to lower frequencies.

Our derived relative sensitivity curve is flat for low-spatial frequencies and decreases for frequencies greater than about three cycles per degree. Hoekstra *et al.* also calculated a single relative-sensitivity curve which they considered to include the influence of number of cycles. Their curve is also flat for low spatial frequencies and decreases for frequencies greater than about 8 cycles per degeree. (Their curve was derived from data taken at much greater luminances than ours; this presumably accounts for the difference between our curves.) It should be noted, however, that the derivation of these curves involves spatial-frequency channels that have much narrower bandwidths than actual channels as yet found psychophysically[12] and physiologically.[13]

In summary, other investigators have been aware that the number of cycles influences threshold, but until our work and the independent investigations of Hoekstra *et al.*,[3] no one had pointed out the dominant role played by this parameter in determining the visibility of low-number-of-cycle targets. Hoekstra *et al.* extended their examination to the relationship between luminance and the minimum number of cycles necessary for maximum sensitivity. Our work has extended the examination to show that the shape of the display, the luminance of the surround, and the observer task can be varied without changing the role played by the number of cycles. Furthermore, our work and that of Hoekstra *et al.* suggest that the low-frequency decrease of reported MTF's is due entirely to the decrease of the number of cycles used in determining them.

ACKNOWLEDGMENT

We would like to acknowledge the assistance of John A. Hall, Jr. in setting up and operating the apparatus, as well as being a subject in these experiments.

APPENDIX

An equation for the data

We will now describe some equations that behave in many ways like the data. Because both observers in all four procedures gave results that had similar shapes, we averaged all of the results and thus obtained smoother data for fitting. We wanted to obtain curves that fit the data, using a simplified description of the data as a guide to the equations. The description we used was

(a) For low number of cycles, there is a linear increase of log sensitivity with log number of cycles (see top graph of Fig. 8).

(b) For high frequencies, there is a linear decrease of log sensitivity with log frequency (see second graph of Fig. 8).

(c) The two influences described in (a) and (b) should be combined to yield a smooth curve (see bottom two graphs of Fig. 8).

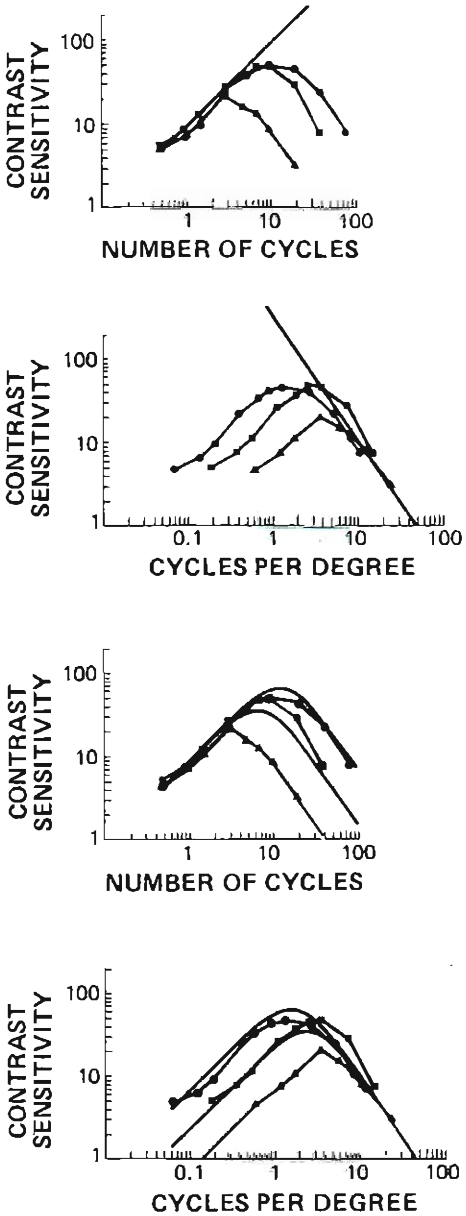

FIG. 8. Straight-line fits to our data in the low-number-of-cycles region (top graph) and high-spatial-frequency region (second graph), respectively. These two straight lines are combined by Eq. (1). The lower two graphs show how well Eq. (1) fits the data.

$$\text{Theoretical Contrast Sensitivity} = \frac{1}{C_1 \cdot (\text{NUM})^{C_2} + C_3 \cdot (\text{CPD})^{C_4}} \,. \tag{1}$$

Equation (1) satisfies the three prescribed conditons. NUM stands for the number of cycles and CPD is the spatial frequency. When the first term in the denominator is much larger than the second, the formula reduces to the general expression for a straight line on log–log paper. The constants C_1 and C_2 are chosen to fit the low-number-of-cycles portion of the data. Similarly, when the second term in the denominator is much larger than the first, the formula reduces to another straight line. The constants C_3 and C_4 are chosen to fit the data at the high-frequency end. Combining the influence of number of cycles and spatial frequency in the manner shown in Eq. (1) is just one way of getting a smooth curve that has the required asymptotic behavior.

The values of C_1, C_2, C_3, and C_4 used in Fig. 8 are 0.12, −1.0, 0.003, and 1.5 respectively. The curves in the lower two graphs of Fig. 8 fit the data quite well. We may note one qualitative difference, however. The equation generates curves that show a shift of the peak on the frequency axis as well as a decrease of the height of the peak as the target gets smaller. This is in agreement with the shape of the 0.83° curves, in which the high-frequency decrease starts influencing threshold before the number of cycles gets large. However, the peak sensitivities of the 2.7° and 7.6° curves are at approximately the same spatial frequency and of the same magnitude.

We have used other types of functions to fit the data. For instance, from the work of Campbell and Robson[7] and Hoekstra *et al.*,[3] it is apparent that the rate of increase of sensitivity with number of cycles is much smaller when the number of cycles exceeds about 5 cycles. A lens would show a much flatter spatial-frequency response for low-spatial frequencies than does the straight line we used to fit the high-frequency end. If we try making the spatial-frequency response flat for frequencies below, say, 3.5 cycles per degree, and also decrease the rate at which the number of cycles influences sensitivity for large-number-of-cycles targets, then we can obtain equations that fit the data slightly better, in the sense that the peaks of the 2.7° and 7.6° curves are more nearly equal.

Fourier analysis of the targets

Our no-plateau targets can be defined

$$L(x,y) = 9.3 + A \cdot \cos[2\pi f(x+w/2)], \quad \text{for } |x|, |y| < w/2, \qquad L(x,y) = 9.3, \quad \text{otherwise.} \tag{2}$$

$L(x,y)$ is luminance in cd/m^2 as a function of position in degrees. Equation (2) represents a uniform luminance of 9.3 cd/m^2, except on a square centered at the origin and w degrees wide. On this central square, a pattern that is uniform vertically and varies sinusoidally in the horizontal direction has been added to the uniform luminance. The sinusoid has its maximum value at the left edge of the square. The frequency of the sinusoid is f cycles per degree. Hence, there are $w \cdot f$ cycles in the display. The amplitude of the sinusoid is A. This corresponds to a contrast of $A/9.3$.

We will use the conventions specified by Bracewell[14] for evaluating the Fourier transform of Eq. (2). It will be convenient to use two of the standard functions defined by Bracewell, $\text{sinc}(z) = \sin(\pi z)/(\pi z)$ and $\Pi(z) = 1$ for $|z| < 1/2$, and 0 otherwise. In considering the transforms of our targets, let us ignore the uniform luminance, which only adds an impulse function at the origin in the transform domain. Then we can rewrite $L(x,y)$ as the product of two one dimensional functions

$$L(x,y) = X(x) \cdot Y(y) \,, \tag{3}$$

where

$$X(x) = \Pi(x/w) \cdot \cos[2\pi f(x+w/2)] \,, \tag{4}$$

and

$$Y(y) = \Pi(y/w) \,. \tag{5}$$

We will use a superscript* to denote the Fourier transform of a function. Then

$$L^*(u,v) = X^*(u) \cdot Y^*(v) \,, \tag{6}$$

where

$$X^*(u) = e^{-i\pi w f} \cdot (|w|/2) \cdot \text{sinc}[w(u+f)] + e^{i\pi w f} \cdot (|w|/2) \cdot \text{sinc}[w(u-f)] \tag{7}$$

and

$$Y^*(v) = |w| \cdot \text{sinc}(w v) \,. \tag{8}$$

In all further discussion, we will be interested in only the magnitude of the transform

$$|L^*(u,v)| = |X^*(u)| \cdot |Y^*(v)| \,, \tag{9}$$

where

$$|X^*(u)| = (|w|/2) \cdot \{\text{sinc}^2[w(u+f)] \cdot \text{sinc}^2[w(u-f)] + 2 \cdot \text{sinc}[w(u+f)] \cdot \text{sinc}[w(u-f)] \cdot \cos(2\pi f w)\}^{1/2} \tag{10}$$

and

$$|Y^*(v)| = |w \cdot \text{sinc}(w v)| \,. \tag{11}$$

The function $|X^*|$ is a one-dimensional transform; several examples of it have been plotted in Fig. 7. It is not influenced by the vertical extent of the display. An argument analogous to that made with respect to Fig. 7 can be made concerning the significance of number of cycles in the two-dimensional transform $|L^*|$. Again, the result is that the size of the display and not the number of cycles *per se* determines the extent to which spectral energy is concentrated at the nominal spatial frequency of the display.

Given our data and the Fourier transforms of the stimuli, we attempted to calculate a modulation transfer function (MTF) for the visual system using some hypothesis about the way in which the visual system processes sinusoidal stimuli. Keeping in mind the fact that the spectral energy of all of the targets is fairly concentrated near the nominal frequency, we tested the hypothesis that contrast sensitivity (CS) is equal to the product of some simple property of the magnitude of the Fourier transform near the nominal frequency times an MTF. This hypothesis (CS = MTF × Property of Transform) is equivalent to assuming that the threshold for visibility is attained whenever the right-hand side of the equation exceeds a fixed value. If the physiological structure that responds proportionately to some property of the transform near the nominal frequency is interpreted as a channel sensitive in that frequency region, then the assumption of CS = MTF × Property of Transform is that the most-sensitive channel establishes threshold. The MTF should, in this context, be interpreted as a measure of the relative sensitivities of these channels. Of course, these would be channels of the entire visual system and would therefore include the modulation transfer function of the lens.

For each of the 30 stimuli, we calculated various functionals of the magnitude of the transform. These included the maximum value of $|X^*|$, $|X^*|^2$, $|L^*|$, and $|L^*|^2$; the integrals $\int |X^*|$, $\int |X^*|^2$, $\int\int |L^*|$, $\int\int |L^*|^2$ over a fixed-frequency range centered at the nominal frequency; and integrals over a frequency range that was proportional to the nominal frequency. The reason for considering the magnitude squared is that the spectral energy density per unit frequency is proportional to $|X^*|^2$ (or $|L^*|^2$). For contrast-sensitivity (CS) data, we used the average data shown in Fig. 8. Finally, the calculated point in the MTF curve from a specific stimulus is CS divided by the functional of the magnitude of the transform.

The maximum value of $|L^*|$ is the most-plausible functional, if the physiological basis of the frequency channels is a cell whose receptive-field organization is uniform in one direction and sinusoidally varying in the perpendicular direction. This functional yields three distinct curves. Clearly, these curves could not represent a single MTF. The functional that came closest to yielding a single MTF was $\int |X^*| du$, integrated from $0.95 \cdot f$, where f is the nominal frequency of the target. It should be noted that this functional has the property that its value is constant for targets with the same number of cycles. This functional results in three curves that overlap so as to represent a single MTF that is flat for low frequencies and decreases for frequencies greater than 3 cycles per degree.

[1]J. J. McCann, R. L. Savoy, J. A. Hall, Jr., and J. J. Scarpetti, Vision Res. 14, 917 (1974).

[2]A sine wave of limited extent possesses spectral energy over a range of spatial frequencies that includes the nominal frequency. So, strictly, it is incorrect to speak of, for example, a 3-cycle-per-degree sine wave when we mean a truncated sine wave with a nominal spatial frequency of 3 cycles per degree. However, because we will be referring to truncated sine waves throughout this paper, it will frequently be convenient to use the looser terminology.

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[8]We found that, with our apparatus, a 1-cycle target had the same threshold whether the left-hand edge was equal to the average liminance (sine phase) or the maximum luminance (cosine phase) of the sinusoidal region. Kelly (Ref. 9) showed that the edge created by a cosine phase target can determine threshold in the no-plateau condition. In our experiments, however, the edge between the mirror and the tube face never completely disappeared, so subjects could not use the creation of an edge as their principal cue for threshold; they needed to perceive some form.

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[10]The definition of bandwidth used in this discussion is width at half-power. For targets with a fixed w, this bandwidth and the peak value of the transform undergo negligible change as n changes except when n is very close to zero. In that case, overlap with the transform component centered at $-f$ makes a significant contribution. However, there is only a 10% change of the height of the peak of the transform of the $\frac{1}{2}$-cycle target, the lowest number of cycles used in our experiments.

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