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VISIBILITY OF CONTINUOUS LUMINANCE GRADIENTS

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Abstract—A plateau of illumination was modulated with various patterns of gradual change: linear slopes and small numbers of low spatial frequency sinusoidal oscillations. Over the range of parameters tested, the threshold contrast necessary for the detection of these modulations was found to be largely independent of the steepness of the gradient, the frequency of the sinusoids, and the size of the target on the retina. Visibility was found to be a function of the fractional change in luminance across the target (contrast) and the pattern of the modulation (characterized by the number of cycles of sinusoid).

INTRODUCTION

We begin by asking the reader to look around the room and find places of uniform reflectance and non-uniform illumination. In particular look for a situation where the illumination must be changing gradually with respect to distance. For example, if your room is illuminated by lamps, look at the wall near a lamp and follow the wall to a greater and greater distance from the lamp. You will immediately find situations in which objects cast shadows, and changes in illumination are clearly visible. However, we are interested in the gradual changes in illumination that you do not see. If you calculate the change in the flux at various distances from the lamp, or if you measure the flux coming to your eyes, you discover that substantial changes in flux go unnoticed by the visual system provided that the changes are gradual (O'Brien, 1958; Cornsweet, 1970; Land and McCann, 1971; Ratliff, 1972). The experiments in this paper attempt to describe quantitatively the physical parameters of luminance gradients at the threshold of visibility.

There is considerable literature concerning the threshold for visual detection of an edge. Blackwell (1946) measured the smallest increment in energy that could be detected against a background. His measurements included various stimulus luminances B_s, various surround luminances B_0, and various sizes of stimuli. Blackwell found that above 10 ft-L the smallest detectable contrast $[(B_s - B_0)/B_s]$ was equal to 0·003 for various size spots. His study included experiments with 6°, 2°, 0·3° and 0·01° spots. Taylor (1964) extended Blackwell's data to include edges larger than 6° and Guth and McNelis (1969) extended the results to include targets with complex shapes such as parallel bars, Landolt rings, printed letters, and dot patterns. All of the above experiments found the limit for the contrast threshold above 10 ft-L to be approximately the same value.

Along another line, numerous investigators (Schade, 1956; Westheimer, 1960; DePalma and Lowry, 1962; Campbell and Green, 1965; Campbell and Robson, 1968; Davidson, 1968; and Kelly, 1960, 1970) have studied the visibility of sinusoidal changes in luminance. Although these targets change gradually from their maximum to their minimum value, they contain many repetitions of a particular gradient. The studies showed that the minimum contrast necessary for seeing a sinusoidal target depends on its spatial frequency. DePalma and Lowry (1962) showed that for the most visible spatial frequencies, the contrast threshold is approximately the same as the threshold described above for edges.

Despite these experiments on abrupt changes and gradual repetitious changes in luminance, the kinds of illumination gradients found in ordinary viewing conditions are relatively unexplored. We are interested in the detectability of a small luminance increment when a single transition occurs gradually instead of abruptly. This paper describes experiments designed to study the interplay of the magnitude of the luminance change with the rate of luminance change on the retina. The results of these experiments led us to perform additional experiments with sinusoidal targets containing from 0·5 to 3 cycles.

METHODS AND MATERIALS

The gradient experiments

Targets. The stimuli for these experiments were square targets whose reflectance changed along one axis but maintained a constant reflectance along the perpendicular axis. We characterized the different targets by luminance measurements along the axis of reflectance change. L_{max} is the highest luminance and L_{min} is the lowest luminance in the target. We used two terms to specify a particular stimulus, *contrast* and *retinal gradient*. Within the study of visual thresholds there are two generally used definitions of contrast. Blackwell defined contrast as $(B_s - B_0)/B_s$ for circular spots on a background, while Kelly (1960) and Campbell and Green (1965) defined it as $(L_{max} - L_{min})/(L_{max} + L_{min})$ for sinusoid targets. We began by comparing our results with those concerning discontinuous edges, so contrast was defined analogous to Blackwell's definition, and is given by $(L_{max} - L_{min})/L_{max}$.

We chose the quantity retinal gradient to describe the rate of change of flux on the retina. It is dependent on both contrast and spatial frequency on the retina. Therefore retinal gradient is proportional to cycles per degree only for targets of the same contrast. Retinal gradient refers to the image of the target on the retina and is given by (contrast)/(retinal angle between L_{max} and L_{min}).

The targets were prepared by placing photographic print paper on an easel near a fluorescent tube that was very long, relative to the width of the paper. Thus, when two corners of the paper were the same perpendicular distance from the tube, all points along the edge between those two points received the same illumination. This insured that one direction of the target maintained a constant reflectance value. Different contrasts were made by rotating the plane of the photographic paper and by adjusting the distance from the lamp to the paper. Further control of the contrast was achieved by proper choice of the print papers and developers. The targets were mounted on a 30·4 cm square black card that had a 7 per cent reflectance. All targets had a reflectance of 50 per cent at the center of the gradient.

Illumination procedures. Each target was viewed in an illumination box (Fig. 2). This box was 90 × 60 × 60 cm with a white interior and a black exterior. The target was placed in a square hole in the back of the illumination box and was held in place by a hinged door. Four 20 W fluorescent lamps illuminated the interior of the box. In addition, two strobe lamps were mounted near the fluorescent lamps for a control experiment in which a 0·15 msec flash of illumination was brief enough to eliminate effects due to eye movements. For these flash experiments, a light projected through a pinhole in the center of the targets was used as a fixation point. All lamps were mounted on the same wall as the target but separated by a baffle so that no light from the lamps fell directly on the target. All of the light falling on the target was reflected from the walls of the box making the effective light source large and the illumination uniform. A uniform reflectance paper was placed in the illumination box and measured with a telephotometer. The maximum variation found due to illumination was 0·007 (computed as contrast).

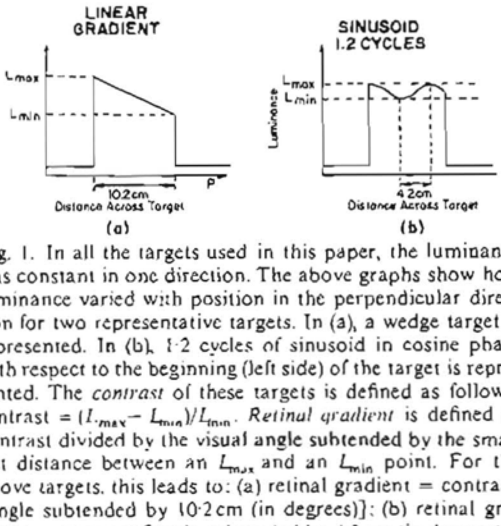

Fig. 1. In all the targets used in this paper, the luminance was constant in one direction. The above graphs show how luminance varied with position in the perpendicular direction for two representative targets. In (a), a wedge target is represented. In (b), 1·2 cycles of sinusoid in cosine phase with respect to the beginning (left side) of the target is represented. The *contrast* of these targets is defined as follows: contrast $= (L_{max} - L_{min})/L_{max}$. *Retinal gradient* is defined as contrast divided by the visual angle subtended by the smallest distance between an L_{max} and an L_{min} point. For the above targets, this leads to: (a) retinal gradient = contrast/[angle subtended by 10·2 cm (in degrees)]; (b) retinal gradient = contrast/[angle subtended by 4·2 cm (in degrees)].

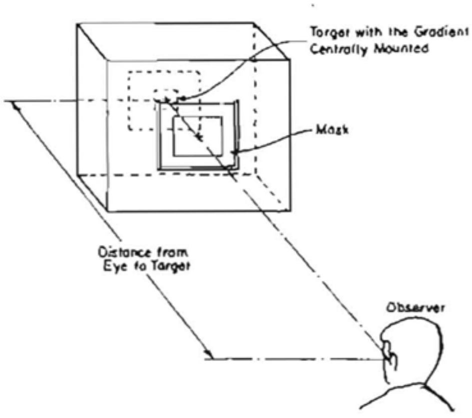

Fig. 2. This diagram shows the arrangement of the observer, the illumination box with its mask and the target. Four fluorescent lamps and two strobe lamps are mounted on the same wall as the target but separated by a baffle. The light from these lamps reflects off the walls of the box and illuminates the target. The mask restricts the observer's view to the target and prevents him from seeing the lamps or the white walls of the box.

The observer looked through a square hole in the face of the wall opposite the target. A mask fitted over this hole allowed the observer to view only the target and none of the inner white walls. During experimentation the room was darkened and the target was the brightest object in the field of view. In all experiments except one, the observers viewed the target binocularly at eye level, using their natural pupils. The exception was a control experiment using a 2·3 mm artificial pupil and monocular vision.

The targets were measured in the illumination box with a scanning telephotometer. The luminance of the center of each target was 154 ft-L. Contrasts for all gradient targets are listed in Table 1.

Experimental procedure and subjects. In the gradient experiment contrast and retinal gradient were varied with five different targets all 10·2 cm². To test whether retinal gradient was the controlling factor, we computed five distances such that the retinal gradient was the same for the first target at the first distance, the second target at the second distance, and so forth. The retinal gradient for Target E at 122 cm is 0·07 and it was this value that was used to calculate the other four distances. For completeness we then tested all targets at all distances.

In all experimental conditions the targets were placed in all four possible orientations and the observers were asked to identify the lightest edge of the square. Since the observer was forced to choose either up, down, left, or right he had only a 25 per cent chance of guessing the correct orientation. Twelve observers viewed each target 16 times from each distance.

The sinusoidal target experiment

Targets, experiments and subjects. For this experiment we prepared seven octagonal targets, 4 cm on a side. We chose the octagonal shape so that we could continue to use a four-alternative forced-choice procedure; the subjects were asked to identify the orientation of the gratings from four possible orientations. Again, reflectance was constant in one direction, but in the perpendicular direction the reflectance varied sinusoidally [see Fig. 1(b)].

Table 1.

Linear targets		Sinusoid targets		No. of
Targets	Contrast	Target	Contrast	cycles
A	0·08	F	0·10	0·5
B	0·12	G	0·11	0·7
C	0·17	H	0·10	1·0
D	0·23	I	0·11	1·2
E	0·33	J	0·10	1·7
		K	0·10	2·0
		L	0·10	2·8

This is a listing of the photographic targets used for the various experiments described in this paper. The second and fourth columns list the contrast $(L_{max} - L_{min})/L_{max}$. Variation in the direction perpendicular to the gradient was measured using a photometer which averaged over the width of the target. In all cases the contrast of this variation was less than 0·025. Targets A–E were square targets while F through L were octagonal. All targets measured 10·2 cm between opposite sides. Targets A through E were mounted on pieces of black matt board 30·4 cm square. Targets F–L were mounted on octagonal pieces of the same material.

The targets were made by photographing a display on an oscilloscope. The oscilloscope display was produced by a technique similar to that used by Campbell and Green (1965). The horizontal sweep of the oscilloscope was set at 1 msec/cm. A high frequency signal from an external oscillator was the vertical input. This signal was given sufficient amplitude and frequency to produce a uniformly bright tube face. A second oscillator, set at a low frequency, was used to modulate the uniform tube face sinusoidally by varying the control grid voltage (Z-axis) of the cathode-ray tube. A section of the tube face was chosen for uniformity, then photographed.

Unlike the gradient experiments, contrast was constant for each target. Each stimulus in this experiment is characterized by two terms: the absolute number of cycles present in the target and the retinal gradient. Retinal gradient is proportional to cycles/deg in this experiment because contrast is fixed. Table 1 lists the contrast for targets F–L in this experiment. As in the initial gradient experiment we first measured the visibility of the seven targets at a single distance (122 cm). Then we calculated the retinal gradient for a half-cycle of the 2·8 cycle target at 122 cm. We then calculated six distances, one for each of the other six targets, so that they had the same retinal gradient. Eight observers made 16 observations of each of five targets at five distances. Two additional targets were run at seven distances. In the sinusoid experiments all targets were displayed in the illumination box under steady fluorescent lighting and the measured median luminance was 152 ft-L.

RESULTS

The gradient experiments. We studied the visibility of luminance gradients as a function of two variables, contrast (the fractional change in the luminance of a target) and retinal gradient (the rate of that change on the retina). See Fig. 1(a). Our measure of visibility was the per cent correct in a four-alternative forced choice procedure where the subject was asked to identify the direction of the gradient. The stimuli used were targets A, B, C, D and E of Table 1. Their contrasts increase from 0·08 for target A to 0·33 for target E. When

Table 2. Visibility of linear gradients

Target	% Correct viewed at 122 cm	Viewing distance such that retinal gradient = 0·07 (cm)	% Correct viewed when retinal gradient = 0·07
A	41	489	48
B	47	222	55
C	68	234	58
D	80	180	77
E	93	122	93

This table lists the results of the five linear gradient targets used for the first experiments. Targets A through E increase in contrast and increase in visibility when viewed at a single distance (122 cm). The third column lists the distances calculated for each target that will generate on the retina a single retinal gradient equal to 0·07. The last column lists the per cent correct when each target is viewed at the distance for the 0·07 retinal gradient. The correspondence of the second column and the fourth column demonstrates that visibility is not determined by retinal gradient.

viewed from a distance of 122 cm these targets have retinal gradients which increase from 0·018 for A to 0·07 for E. As shown in column 2 of Table 2, there was a marked increase in visibility as we progressed from target A to E at that viewing distance. Since retinal gradient is proportional to contrast for these targets at a fixed distance, this experiment alone does not allow us to distinguish between the effect of changing contrast as opposed to changing retinal gradient. We tested whether retinal gradient was the determining factor by viewing targets with different contrasts but with the same retinal gradient. This was accomplished by viewing each target at a different distance. Since retinal gradient is contrast divided by the angle between L_{max} and L_{min} we can calculate a distance for each target so that the retinal gradient equals a constant. This is equivalent to saying that at these specific distances there is a constant rate of change of flux with respect to distance on the retina for each target.

If visibility depended only on retinal gradient, then all the targets should have been equally visible. This was not the case: each target had a different visibility. What was more interesting each target had approximately the same visibility as it did when it was viewed at 122 cm (compare column 2 with column 4 of Table 2). This result suggested that contrast and not retinal gradient correlated with visibility.

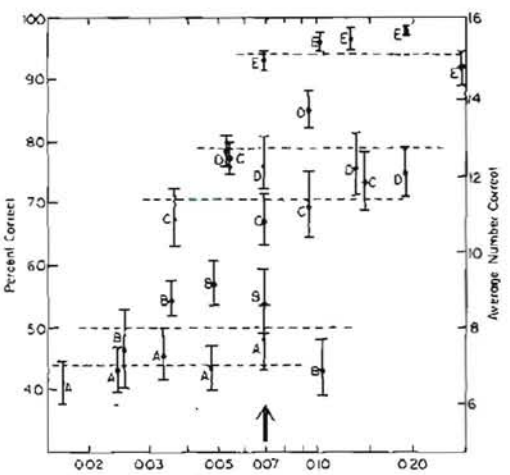

Fig. 3. This graph summarizes the results of the experiments with the gradient targets. Retinal gradient is plotted against the per cent of correct identification of the direction of the gradient. Each target, A–E, was viewed at five distances and hence had five different retinal gradients. For any one target the smallest retinal gradient is associated with the smallest distance between observer and target, and the largest subtended visual angle. The distances were calculated so that each target had the same retinal gradient (0·07, see arrow) at some distance. When the retinal gradients were identical, visibility was a monotonically increasing function of target magnitude. The graph shows the mean per cent correct ± 1 S.E. for each target at each distance. The horizontal dashed lines are the average of the 5 means for a single target. These averages are a fair fit to the data in the sense that each target has approximately the same visibility independent of viewing distance.

We then asked the observers to view all five targets at all five distances. If visibility of a gradient depends only on the contrast, then we would expect any particular target to be equally visible at all viewing distances. For a given target, increasing distance corresponds to increasing retinal gradient. Figure 3 is a graph of the per cent correct vs the retinal gradient for this experiment. The mean ± 1 S.E. of all 25 distinct target–distance presentations are shown. For each target–distance measurement each of 12 observers made 16 observations. For each distance the visibility of the targets increased with contrast. Furthermore, each target had approximately the same visibility at all distances. The horizontal dotted lines through Fig. 3 show the averages for all results for each target. We used the standard error of estimate to determine how well the horizontal line fits the observers' results. On the average, observers identified the direction of target A 7·1 times in 16 attempts and the standard error of estimate was 2·2. The other results were: target B, 8·2 ± 2·6; target C, 11·5 ± 2·4; target D, 12·6 ± 2·4; target E, 15·3 ± 1·3.

Within the limits of this experiment, it was not possible to make a continuous wedge more visible by changing the distance between the target and the observer. Despite variation in slope on the retina by a factor of 4, the visibility of these targets remained essentially unchanged. This idea would have interested the Gestalt psychologists as another example of visual constancy. These results are a little disturbing when one recalls the data showing that the threshold contrast for the visibility of sine waves depends upon spatial frequency. (For a fixed contrast, retinal gradient is proportional to spatial frequency.)

The first explanation might be that we have overlooked some subtle variable effect and need additional control experiments. We tested whether threshold visibility was determined by time dependent comparisons. One explanation of our results might be that the eye moved quickly from one side of the target to the other, so that receptors could read luminances separated by time instead of distance. In this manner the total change across the target could be detected independent of the retinal gradient. We used a brief strobe illumination (0·15 msec) to prevent motion of the stimulus on the retina. It was much harder to make a judgment with such a brief flash, but target D, which was less than 100 per cent visible in the original experiment remained well above the chance level of visibility with strobe illumination. Table 3 shows the results of viewing this target at three distances. The target was less visible than in the original experiment (45 per cent correct instead of 76 per cent), but the visibility was unaffected by changing the distance, and hence was independent of retinal gradient.

The other control experiment tested whether variations in size of the natural pupil affected our results.

Table 3. Visibility in flash illumination

Distance observer and target (cm)	% Correct in flash experiment	% Correct in the first experiment
122	44	77
234	44	74
489	46	76

This table describes the results of the flash experiment. Only Target D, described in Table 1 was used in this experiment. The right hand column is a measure of the visibility of this target by the same observers under the continuous illumination of the first experiment. Although there is a marked decrease in the visibility of the target, there is no change as the angle subtended is changed.

Since the targets subtended markedly different angles at different distances, the total amount of light energy entering the eye changed with the distance. Such changes would affect the size of the pupil. In this control experiment we had a single observer view each of three targets 96 times at each of three distances. He viewed the targets monocularly, using a 2·3 mm artificial pupil. His results are listed in Table 4 and show that the addition of the artificial pupil has no effect on the results found in the first experiment. The visibility of a given target is still independent of distance with the exception that target D at the closest distance is unexpectedly less visible than at the other distances.

Table 4. Artificial pupil experiment

Target	Per cent correct at 122 cm (%)	234 cm (%)	489 cm (%)
B	45	54	52
C	58	67	69
D	57	84	79

The above table lists the results of one observer who viewed three targets at three distances through a 2·3 mm artificial pupil in continuous illumination. Each percentage is based upon 96 observations.

Sinusoid experiment. For the linear gradients, contrast was found to be the principal determinant of visibility. This was surprising because work with sinusoidal gratings has demonstrated that visibility is dependent upon a variable analogous to retinal gradient, namely spatial frequency. Later in the paper we will examine in detail the implications of the differences between our targets and conventional sinusoid gratings as used by DePalma and Lowry (1962) and others. But first, let us experimentally explore the relationship between our linear gradient targets and analogous sinusoidal targets.

A half cycle of cosine is similar to our wedges in that its luminance changes monotonically from side to side, and it can be given an amplitude such that its retinal gradient and contrast are the same as those of a target with a linear slope. We would expect that the visibility of a half cycle cosine target will not depend on the frequency of the cosine, since that frequency corresponds to retinal gradient which was found to be unimportant in the first set of experiments.

Starting from our monotonic gradients we moved toward sinusoidal targets by using targets which contained only a small number of cycles (ranging from 0·5 to 2·8 cycles). All seven of these targets had the same contrast and were viewed at 122 cm. Table 5 shows that the half cycle target was correctly identified 22 per cent of the time while the 2·8 cycle target with the same contrast (0·10) was identified 100 per cent of the time. As the number of cycles increased from 0·5, the visibility increased monotonically from 23 per cent correct until at 2·8 cycles the target was 100 per cent visible. Since each target was the same size, a half cycle of the 2·8 cycle target subtended a much smaller angle than the 0·5 cycle target and hence had a larger retinal gradient. We repeated the procedure used in the first part of the paper to determine whether the visibility of sine waves was also independent of retinal gradient. Target L, viewed at 122 cm, gave a retinal gradient of 0·12. A viewing distance was calculated for each target F–K, so that at that distance that target had a retinal gradient equal to 0·12. These distances and the per cent correct at these distances are listed in Table 5. In addition, the last column of Table 5 lists the averages of per cent correct for each target at all distances tested.

At first glance there seems to be no significant difference between viewing all the targets at 122 cm, viewing them at different distances so they have identical retinal gradients, and the average of viewing them at many distances. Figure 4(a) is a graph of per cent correct vs retinal gradient (or cycles/deg) for each target at each distance. The dashed lines are the averages of the results for each target over all distances. Each target has a distinct visibility that is largely independent of retinal gradient. The 0·5 cycle target (at chance) and the 2·0 and 2·8 cycle targets (at complete visibility)

Table 5. Visibility of sinusoid targets

Target	Cycles	% Correct at 122 cm (%)	Distance for retinal gradient = 0·12 (cm)	% Correct at this distance (%)	% Correct at all distances (%)
F	0·5	22	749	19	23
G	0·7	30	472	28	24
H	1·0	60	368	75	67
I	1·2	70	274	75	83
J	1·7	86	221	86	83
K	2·0	95	196	100	99
L	2·8	100	122	100	100

This table lists the results of eight observers who viewed seven sine wave targets. For each target–distance combination there were 128 observations. The third column lists the per cent correct at 122 cm. As in Table 2, distances were calculated so that each target could be viewed at the same retinal gradient. Target L, viewed at 122 cm, gave a retinal gradient of 0·12 and it was this figure that was used for the calculation of the remaining six distances. Targets H and J were viewed at all seven distances. All other targets were viewed at the five distances other than 368 and 221 cm. The final column lists the average per cent correct for a given target at all distances tested.

show no variation as a function of retinal gradient. The other targets are slightly more visible at higher retinal gradients than at lower ones. Nevertheless, the presence of seven non-congruent curves, one for each target, demonstrates that retinal gradient is not the important variable in determining the visibility of these targets.

All of these targets have the same contrast so that we are left with the number of cycles as the significant variable controlling the visibility of these targets.[1] Figure 4(b) is a graph of per cent correct vs the number of cycles in the targets for the various distances. One could draw seven curves through the data: one for each distance. However, since all seven curves would almost coincide, only one curve was drawn by eye. An exception would have been the curve for the very closest distance, 122 cm. The targets were consistently less visible by a small amount at that distance.

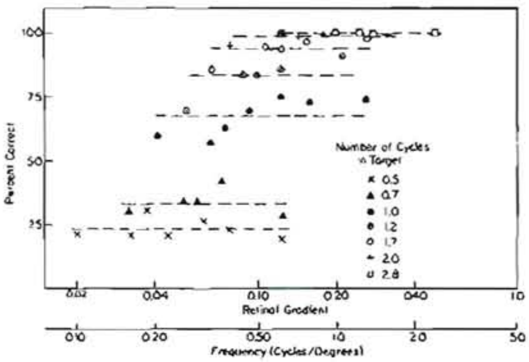

Fig. 4(a). This graph plots per cent correct versus retinal gradient for each sinusoid target at each distance. Each target had a different number of cycles. All targets had very nearly the same target magnitude, and therefore spatial frequency was proportional to retinal gradient. Retinal gradient for a particular target was varied by changing the distance between observer and target. Larger distances correspond to larger retinal gradients. The dashed lines are the averages of the results for one target over all the distances. Each target has a distinct visibility that is largely independent of retinal gradient. The 0·5 cycle target (at chance) and the 2·0 and 2·8 cycle targets (at complete visibility) show no variation with retinal gradient. The other targets when viewed at the closest distance are somewhat less visible than the average. This is illustrated more clearly in Fig. 4(b).

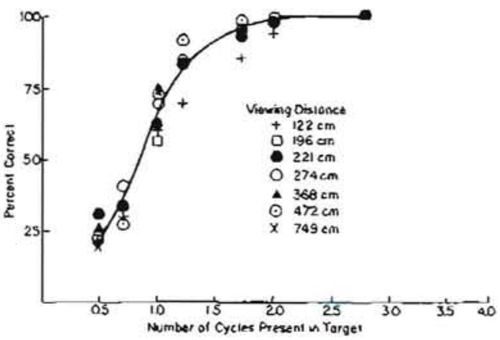

Fig. 4(b). This graph plots the per cent correct versus the number of cycles present in the targets for each target at each distance. One could draw seven curves through the data: one for each distance. However, since all seven curves would almost coincide, only one curve was drawn by eye. An exception would have been the curve for the very closest distance, 122 cm. The targets were consistently less visible by a small amount at that distance.

SINUSOID: VARIABLE CONTRAST EXPERIMENT

We began this study by trying to measure how large a gradual change of illumination was visible. Intuitively, we assumed that the rate of change of flux on the

[1] Campbell and Robson (1968) mention that the threshold contrast necessary for seeing their sinusoidal targets of low spatial frequency increases when the number of cycles presented goes below about 4.

retina would be among the most important variables. Our results showed that rate of change of flux, which we called retinal gradient, had almost no effect on visibility. For linear gradients the only variable that influenced the visibility was contrast.

While pursuing this unexpected result we studied the visibility of different number of cycles of sinusoidal gradients with the same contrast. We found that a 1·7 cycle target was nearly 100 per cent visible while a 0·5 cycle target with the same retinal gradient was only 25 per cent visible (chance). Whatever the mechanism involved in detecting gradual changes, it must now explain why two and three identical gradients connected together to make a 1·0 and 1·5 cycle target are much more visible than a single gradient by itself. In these sinusoid experiments we have not varied the parameter we found most important in the wedge experiments, namely contrast. The final set of experiments studies the visibility of these targets when both contrast and number of cycles are varied using a single distance between observer and target.

METHODS AND MATERIALS

Two subjects were used. Each subject viewed each target 64 times in a four-alternative forced-choice procedure. Their task was to state the orientation of the stripes in the sinusoidal targets. Subjects placed their heads in a pair of head rest goggles that determined the position of the head and occluded the left eye. The targets were created by adding two sources of illumination. One was a box designed to provide a uniform illumination across the central portion of a partially silvered mirror. This box was a smaller version of the illumination box used in the previous experiments. Behind the mirror the spatially varying part of the target was generated on the cathode ray tube of 535A Tektronix oscilloscope. The horizontal and vertical intensity inputs to the scope were obtained from a device described in detail below. A mask was glued to the silvered surface of the mirror. It was shaped so that the unmasked portion looked like a regular octagon when the mirror was viewed at a 45° angle. The targets were approximately 5 cm from side to side. Observers viewed the targets from a distance of 89 cm. This combination of target size and distance corresponded to the second closest distance used in the previous sinusoid experiments in the sense that the targets subtended the same visual angle.

Subjects were instructed to close their eyes while the targets were being rotated because observers reported that targets seemed especially visible when they were changed. A colored filter was placed between subject and mirror so that the entire display appeared to be of one color (green). This was necessary because the uniform illumination was white while the tube signal was light blue. The average illumination in the targets was approximately 7 ft-L. Measurements of each target were made with a scanning telephotometer.

We built a device that switched the sinusoidal displays on the oscilloscope in four different orientations at any desired phase. This device allowed the experimenter to switch rapidly and easily from one orientation to another. To go from a horizontal to a vertical display, the device would simply switch the X and Y inputs. For example, if $g(t)$ is the sweep function (sawtooth) and $f(t)$ is the triangle raster, then $g(t)$ goes into X and $f(t)$ goes into Y thus generating a horizontal raster which can be modulated by a sinusoidal Z-axis input (intensity of electron beam). To get a vertical display, just send $f(t)$ to X and $g(t)$ to Y. To get a diagonal display, we need $[f(t) + g(t)]$ going to X and $[f(t) - g(t)]$ going to Y. This, however, would give a display which is $\sqrt{2}$ longer than the horizontal target. So we need $[f(t) + g(t)]/\sqrt{2}$ going to X and $[f(t) - g(t)]/\sqrt{2}$ going to Y. To generate these functions operational amplifiers were needed. One of the biggest technical difficulties was obtaining amplifiers which had less than 1° phase shift up to 100 kHz. Such operational amplifiers were necessary because the raster would form Lissajous figures near the edges of the display if there was even a very small phase shift. These edge phenomena were made as small as possible by using the appropriate amplifiers. Furthermore, because of the octagonal mask, only the central portion of the display was actually used in the experiments.

The next consideration was the phase of the sinusoidal target. The device had to synchronize the beginning of each sweep with any point on the sine wave coming from a Hewlett–Packard 201C audio oscillator. A circuit was used which is analogous to the usual triggering apparatus available with oscilloscopes. It scans the input signal until a certain slope is obtained and then the sweep begins. Thus, we could display one cycle of sine wave or cosine wave or any phase in between.

RESULTS

The results are presented in Fig. 5(a). The per cent correct is plotted against the number of cycles for four sets of targets, each set with a different contrast. It is interesting that over this range of contrasts each set of targets exhibits a dramatic increase in visibility in the region of 0·5–1·5 cycles. Despite this rapid change of visibility over a small range of number of cycles, at any particular value of number of cycles the greater the contrast the greater the visibility.[2]

To clarify the interplay of contrast and number of cycles in determining visibility we have presented the results of the experiment in a different form. The graphs of Fig. 5(b) are obtained from the upper graphs by linear interpolation between experimental data points and extrapolation to the points where the curves of the upper graphs just reach the 100 per cent visible and chance visibility boundaries. The lines in

[2] If one looks back at Fig. 4(b) one sees that those photographic targets, all of which had a contrast of 0·1, were less visible than the 0·1 contrast stimuli as presented on the oscilloscope. We think there are two reasons for this. First, the average level of illumination was about 150 ft-L in the Fig. 4(b) data as opposed to 7 ft-L for the oscilloscope targets. To check whether this increase in average illumination made the targets less visible, observers RLS and JAH viewed the photographic targets through a 1·3 neutral density filter which effectively reduced the luminance by a factor of 20. The targets were found to be slightly more visible. The second reason for the greater detectability of the oscilloscope targets was practice. Practice is known to decrease threshold for sinusoid targets (Davidson, 1968). The two observers received much more practice with this type of target than the observers in Fig. 4(b) had.

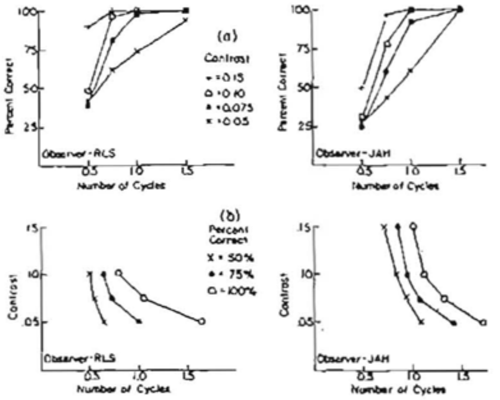

Fig. 5. These graphs show the individual results of the two observers. Each point represents 64 observations. (a) In the upper graphs per cent correct is plotted as a function of number of cycles. Each curve represents the visibility of targets of a particular target magnitude. These target magnitudes are specified in the legend between the graphs. It is clear that both target magnitude and number of cycles influence the visibility of the targets. (b) In the lower graphs the same data is replotted on different axes to clarify the interplay of the two variables. These graphs were obtained from Fig. 5(a) by linear interpolation between experimental data points and extrapolation to the points where the curves of the graphs just reach the 100 per cent visible and chance visibility boundaries. The lines in the graphs are lines of constant per cent correct.

the graphs are lines of constant per cent correct. Visibility is clearly dependent upon both the number of cycles and the contrast.

[3] As with most constancy phenomena, these have their limitations. Figure 4(b) suggested that there is a slight decrease in visibility of the sine wave targets at 122 cm. Also, one might ask whether all the data fits the horizontal lines in Fig. 2. Perhaps the lack of perfect fit hints at the existence of small effects due to distance or size on the retina or retinal gradient which our measurements by themselves cannot specify.

[4] The plateau is necessary if we wish to examine the dependency of visibility upon number of cycles independent of the phase of the stimuli. Kelly (1970) showed that this is a crucial consideration in the detection of low frequency gratings. His sinusoidal targets were modulations of a portion of a uniform background. What made phase important in that situation was the creation of a discontinuity (edge) for phases other than 0°. In particular, a 90° phase shift gave the largest discontinuity and the most visible target. This is a situation where edge effects, rather than frequency, is the critical factor.

The situation with our targets was quite different. We were modulating the top of a plateau of illumination, so there was always a large visible discontinuity. We chose the phase of the sine wave to be 90°, a cosine, so that the maximum variation would be present with only 0·5 cycle. We could safely make this choice because of the large discontinuity which was present in any case.

DISCUSSION

Let us begin this discussion by summarizing the results of the three main experiments. It should be understood that these conclusions, simply stated, are meant to apply only to the experimental conditions already discussed. The first set of experiments showed that the visibility of a linear gradient was dependent on contrast and not retinal gradient. The second set of experiments, using sinusoidal targets of fixed contrast, showed that visibility depended upon the number of cycles. The third set of experiments showed that the visibility of sinusoidal targets depended on both contrast and the number of cycles. These results can be alternatively described by the following two statements. First, the visibility of a particular target was essentially constant independent of the viewing distance, hence largely independent of size and rate of change of energy on the retina.[3] Second, the spatial pattern of the target, usually described in these experiments as the number of cycles, can be as important as the contrast.

In no case did we find that retinal gradient was the dominant variable controlling visibility. This was surprising because work with sinusoidal gratings has demonstrated that visibility is a function of a variable analogous to retinal gradient, namely spatial frequency. (For a fixed contrast, retinal gradient is proportional to spatial frequency.) For example, DePalma and Lowry (1962) studied the threshold contrast for sinusoids of various frequencies while varying the distance between observer and target. Not only did they find the high-frequency and low-frequency threshold increases that others have reported, but they also showed that the form of the threshold vs spatial frequency curves varied only slightly with distance. This indicates that it is truly the frequency on the retina which is the crucial variable in determining the visibility of their targets.

Of course, there are important differences between the targets used in our experiments and those used by DePalma and Lowry (see McCann *et al.*, 1973). Their stimuli were modulations of an entire field of uniform luminance. Ours were modulations of a luminance plateau which was surrounded by a uniform black area.[4] Their modulations consisted of many cycles of sinusoidal variation of luminance with position. Our modulations consisted of a single linear transition or of a small number of sinusoidal oscillations from one side of the plateau to the other.

The threshold for a 0·6 cycle/deg grating (3·6 cycles viewed at 89 cm) as found by DePalma and Lowry is approximately 0·01 (expressed in terms of contrast). Our data in Fig. 4 show that 0·5 cycles at that spatial frequency is invisible at a contrast of 0·1. At the same contrast and frequency 1·0 cycles is 75 per cent visible and 2·0 cycles is 100 per cent visible. Thus, we are operating in a region well above the threshold as found by DePalma and Lowry. Apparently, the presence of a plateau or the small number of cycles involved or both has created targets which are more difficult to see in the sense that they require a greater contrast to be visible. Spatial frequency is no longer the threshold-setting variable as it was for DePalma and Lowry's targets. Instead, the pattern of the target becomes the crucial factor in determining the minimum contrast necessary for visibility.

There are several intriguing hypotheses that make use of the target's spatial pattern properties found in these experiments. The linear gradient experiments demonstrated that the size of the change from one side of the target to the other was the critical variable that corresponded with visibility. One could hypothesize that the mechanism that controls visibility of linear and 0·5 cycle sine wave targets need only be sensitive to the size of the discontinuities at the edges of the target in order to calculate the contrast. It could be argued that the difference in the magnitudes of the discontinuities on the two opposite sides was the key piece of information which the visual system used to detect these targets. In fact, such a mechanism would account for the constant visibility of these targets despite large changes in retinal gradient. The visual system might compare the ratios of energy at the edges and then use any difference in the ratios to detect the contrast of the targets. The visibility of sine wave targets is dependent on both contrast and number of cycles. Determining the contrast of the target, using edge ratios or any other means, will not account for threshold visibility since it varied with number of cycles for a single contrast. Thus the comparison of ratios of energies at edges is not a sufficient mechanism to detect these targets.

Another model which can make use of the targets' spatial pattern involves the modulation transfer function (MTF). The linear systems analysis approach as applied to experiments by Campbell, Davidson, Kelly and others, is a general method for obtaining a lightness distribution from a given luminance distribution. It has been used with success to account for the existence of light and dark Mach bands where there is a gradual transition region between a uniform light area and a uniform dark area.

It is ironic that experiments very similar to those which gave support to the linear systems analysis approach are also the source of one of the objections to it. Consider a sequence of brightness distributions progressing from the Mach band generating pattern described above to a pattern which has just the uniform low and high regions with a sharp edge between them. As we progress along the sequence, the central changing region gets narrower and steeper until it becomes the edge discontinuity. The MTF model predicts the existence of light (and dark) bands in the response which get lighter (or darker) and narrower as we progress through the sequence. Even in the case of the edge, despite a discontinuous distribution, the model has no mathematical problems. Well-defined operations take place. Large but finite Mach bands of non-vanishing width are predicted. However, observers do not see such bands. Observers sometimes report extremely narrow bright lines near the edges, but these are much narrower and fainter than the predicted bands. Davidson and Whiteside (1971) discuss this problem in greater detail, but are unable to resolve it within the context of the MTF model.

What is the impact of all this on our experiments? The targets used in the experiments of this paper can be thought of as the sum of two parts. One part is a plateau of illumination of height $(L_{max} + L_{min})/2$ sitting on a black background. In the case of the wedge targets, the second part is a linear gradient which traverses the width of the plateau. In the case of the sinusoid targets, the second part is a truncated cosine wave; that is, a cosine starting at one side of the display and going for as many cycles as it can until it reaches the other side.[5]

One can think of the two components of the target as signal and noise. Since the wedge or cosine wave is what the observers are trying to detect, let's call that the signal. The noise, then, is just that part of the target which the linear systems approach has failed to adequately model. It is the source of most of the target's energy, since the coefficients of the gradients are typically one-tenth that of the plateau. The amplitude of the predicted Mach bands is much larger than the predicted response to any of the signals (see Fig. 6). Yet, observers do not report Mach bands but are able to detect the signals. Such a discrepancy led us to the conclusion that the MTF approach was not the appropriate way to model our experiments.

Of course, the distinction between a mathematical tool and a psychophysical model should be kept in mind. Even if a simple application of the modulation transfer function of the eye does not always correctly predict the observer's response. Fourier analysis of the target supplies useful information that has stimulated new experiments. For example, Carter and Henning (1971) made use of the fact that the energy in one cycle of sine wave at 5·9 cycles/deg is distributed over a wide range of spatial frequencies, whereas the energy of 160 cycles at the same spatial frequency is highly concentrated at the nominal frequency of the sinusoid. Using

[5] Of course, one must be careful in using these mathematically convenient ways of thinking about the targets. For example, the background plateau must have a sufficient intensity that the sum of the two components is never less than zero. In addition, we cannot think of our targets as a sum of two parts when we proceed to the actual MTF calculations in the sense that we cannot consider each part separately. The reason for this, is that the MTF model is linear only after the logarithm of the luminance distribution has been taken (Ratliff, 1965; Whiteside and Davidson, 1971). So, we should do the MTF calculations with the logarithm of our input function. Alternatively, we could present targets which were exponentiated versions of our targets, and use the linear version as the direct input to the MTF model. However, because we are dealing with small perturbations of a simple plateau, these considerations are quantitatively unimportant.

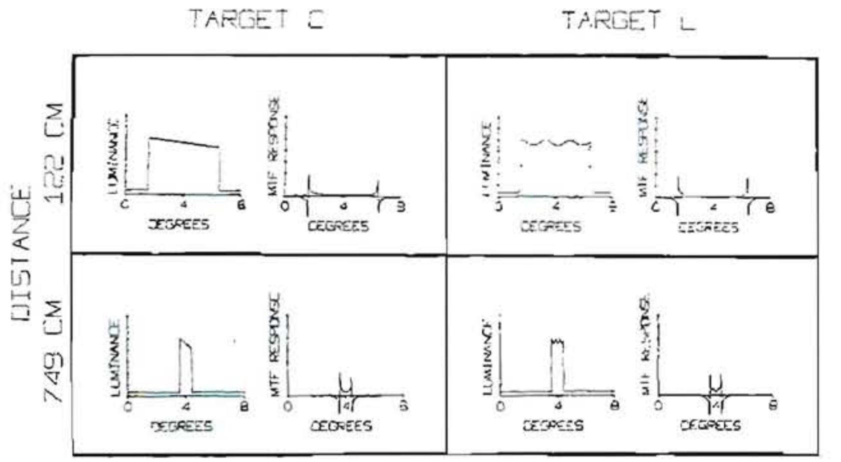

Fig. 6. This figure shows the luminance distribution and predicted MTF responses for targets C and L at the closest and furthest viewing distances. We used the MTF curve reported by Cornsweet (1970, p. 341) with linear extrapolation for very low spatial frequencies. The response predicted by the MTF model is dominated by large Mach band-like effects at edges. These effects are not seen in the targets. For example target L is the bottom left target in Fig. 7. The sinusoidal gradients are clearly visible and the Mach band-like edge effects are not visible. The size of these predicted edge effects is determined by the size of plateau and is independent of viewing distance.

narrow-band and broad-band veiling luminances they showed that the visibility of the single cycle was decreased more by broad-band noise while the visibility of the 160 cycle target was decreased more by narrow-band noise. In the case of our targets, the plateau of illumination can be viewed as another kind of broad-band noise. However, the difference between 0·5 and 1·5 cycles in terms of concentration of energy at various spatial frequencies is very small compared to the difference between 1 and 160 cycles. (The ratio of number of cycles enters the calculations, and 3 is small compared to 160.) Yet, we find an increase in visibility going from chance to 100 per cent correct with this small change in the number of cycles.

The nominal frequency of Carter and Henning's targets was approximately 6 cycles/deg, a frequency generally recognized as being in the optimal region for detection (DePalma and Lowry, 1962; Davidson, 1968). The work of Blakemore and Campbell (1969) presents evidence for the existence of neural units specifically selective to such spatial frequencies and higher frequencies. However, they find no such units for spatial frequencies below about 3 cycles/deg. A glance at Fig. 4 shows that for the targets used in this paper, the nominal spatial frequency is below 3 cycles/deg. Furthermore, if we look at the actual Fourier spectrum of the signals (plateau not included) we find that even though the energy is not localized at the nominal frequency, the integral of spectral energy from 0 to 3 cycles/deg is almost unchanged as we go from 0·5 to 1·5 cycles for our targets. Almost all the energy is in that region.

Changing the number of cycles increases total energy below 3 cycles/deg by a few per cent, but increasing contrast from 0·05 to 0·10 increases the integrated energy in that region by a factor of 4. (We square the Fourier spectrum before integrating.) If we weight the Fourier components using the MTF, there is a larger change in going from 0·5 to 1·5 cycles because the nominal frequency is also increased and we are on the portion of the MTF curve where increasing frequency implies increasing sensitivity. However, calculations show that this is still a small increase compared to doubling the contrast. So, if the small increase in energy going from 0·5 to 1·5 cycles at 0·05 contrast raises visibility from chance to 100 per cent (Fig. 5), then the increase from 0·05 to 0·10 contrast at 0·5 cycles should do at least as much. In fact, Fig. 5 shows that it doesn't and this implies that the increase in visibility with increasing number of cycles is not due to simply exceeding threshold for some frequency detector which integrates energy below 3 cycles/deg.

We have now discussed several models. We are unable to account for all of our experimental results with any one model. Nevertheless, with the three targets in Fig. 7 (all having the same contrast) we can illustrate the two visual properties described by these experiments. First, the fact that the orientation is easier to see as the number of cycles increases illustrates the dependence of visibility on the number of cycles. Second, the fact that viewing the figure at any distance corresponding to the experimental conditions will not substantially change the visibility illustrates the lack of dependence on retinal gradient and the nominal spatial frequency.

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Résumé—On module un plateau d'éclairement par divers types de changement graduel: pentes linéaires et oscillations sinusoïdales de basse fréquence spatiale en petit nombre. Dans le domaine étudié pour ces paramètres, le seuil de contraste nécessaire pour détecter ces modulations est largement indépendant de la raideur du gradient, de la fréquence des sinusoïdes, et de la taille du test sur la rétine. On trouve que la visibilité est fonction de la fraction de changement de luminance à travers la cible (contraste) et du type de modulation (caractérisé par le nombre de cycles de la sinusoïde).

Zusammenfassung—Ein Feld homogener Leuchtdichte wurde mit verschiedenen stetigen Leuchtdichtemustern variiert: Mit linearen Gradienten sowohl wie mit Sinusgittern niedriger Ortsfrequenz. Bei allen untersuchten Parametern wurde gefunden, dass der Schwellenkontrast für die Erkennbarkeit dieser Modulationen weitgehend von der Steilheit des Gradienten, von der Ortsfrequenz des Sinus und von der Grösse des Testzeichens auf der Netzhaut unabhängig war. Die Sichtbarkeit war eine Funktion der relativen Leuchtdichteänderung (Kontrast) und des Modulationsmusters (charakterisiert durch die Zahl von Perioden im Sinusgitter).

Резюме—Ровноосвещенное поле модулировалось различными градально меняющимися паттернами: лнейными градиентами и небольшим числом низкочастотных синусоидальных колебаний. Изменялись параметры модуляции и определялись пороги ее обнаружения. Они оказались в широких пределах независимы от крутизны градиента, частоты синусоиды и величины изображения объекта на сетчатке. Было найдено, что различимость являетя функцией фракционного изменения яркости в пределах объекта (контраста) и паттерна модуляции, характеризующегося числом циклов синусоиды.

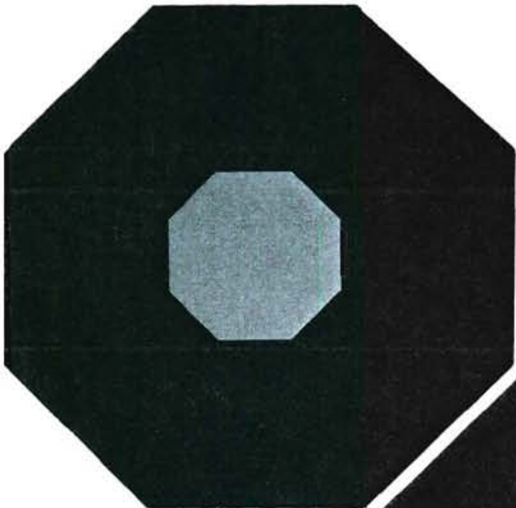
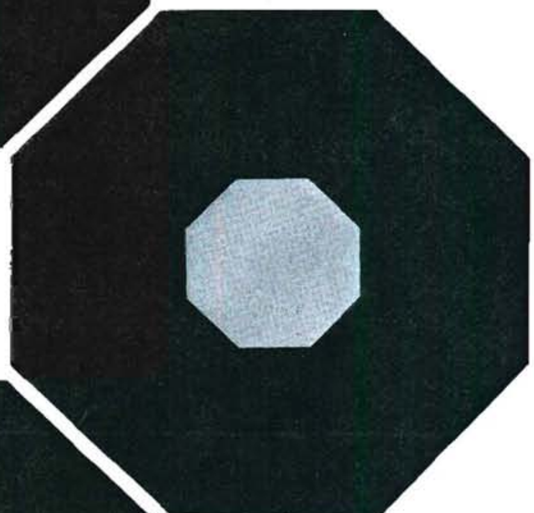
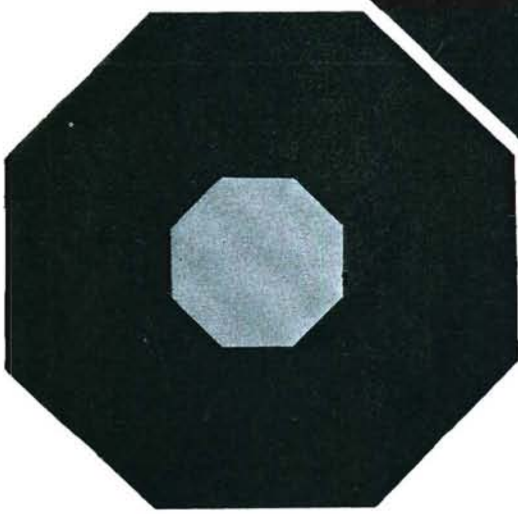


Figure 7. This is a photograph of targets F, J, and L. All three targets have the same contrast. Hold the figure 17 inches from your eyes to duplicate the viewing size used in the oscilloscope experiments and the second distance of the photographic target experiments. Try to select the orientation of the sinusoidal patterns.



The bottom target is 2.8 cycles and it is oriented diagonally such that the dark stripes go from the lower right to the upper left. The target at right is 1.7 cycles oriented diagonally such that the dark stripe goes from the lower left to the upper right. The top target is 0.5 cycles. It is oriented vertically with L_{min} on the left and L_{max} on the right.



The orientation is easier to see as the number of cycles increases, even when the target is moved from 11 inches to 62 inches. At a viewing distance of 11 inches the 2.8 cycle target has the same nominal spatial frequency as the 1.7 cycle target at 18 inches and the 0.5 cycle target at 62 inches.