Vision Res. Vol. 16, pp. 445 to 458. Pergamon Press 1976. Printed in Great Britain.

QUANTITATIVE STUDIES IN RETINEX THEORY

A COMPARISON BETWEEN THEORETICAL PREDICTIONS AND OBSERVER RESPONSES TO THE "COLOR MONDRIAN" EXPERIMENTS

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(*Received* 10 *April* 1975)

Abstract—Land's Color Mondrian experiments showed that a single wavelength-radiance distribution falling on a point on the retina can generate nearly any color sensation. In Part I we repeated that experiment, quantifying the color sensations for each of the many Mondrian areas. In Part II we show that each area's color sensation correlates with a triplet of reflectances measured with photodetectors having the same spectral sensitivities as the cone pigments in the eye. This result provides a description of what the visual system does, but it does not provide a mechanism for how the visual system can do it because the reflectance measurements required the use of a reflectance standard and unchanging illumination. In Part III we describe a model for color sensations that computes three reflectances from the wavelength-radiance distribution without reflectance or illumination standards: hence, it is able to predict the color sensations seen by the observer. The model is able to predict gray, red, yellow, green and blue sensations associated with areas that send identical wavelength-radiance distributions to the eye.

Key Words—Color Mondrian experiments; color sensation; model for color sensations.

In the human eye there are three types of cones, each containing a different photosensitive pigment (Marks, Dobelle and MacNichol, 1964; Brown and Wald, 1964). It is often assumed that the color at each point in the visual image depends only on the relative energies absorbed by these three pigments from the light incident at that point on the retina. In general, this simple approach to color is incorrect. While it is certainly true that any color can be matched by suitable adjustment of the intensities of three fixed primaries, a particular mixture of those primaries does not specify a unique color sensation (Wright, 1972; Wyszecki, 1973). Helmholtz (1924), in his chapter on contrast, cites a variety of observations that show that the color of an area changes when areas adjacent to it are changed. Helson (1938), Evans (1948) and Albers (1963) extended these observations experimentally. Land (1959a, 1962, 1964, 1975) showed that in moderately complex images there is no unique color sensation associated with a particular wavelength-radiance distribution at a point. Land's "Color Mondrian"[2] experiments demonstrate that a particular wavelength-radiance distribution can produce nearly any color sensation.

The Color Mondrian display, used in those experiments, consisted of about 100 different colored matte papers arranged arbitrarily so that no particular color surrounded another. In fact, each paper was surrounded by at least five or six different colored papers (see Land, 1975, for color photograph of display). The display was illuminated by three projectors, each with a different interference filter. One filter transmitted part of the long-waves of the spectrum, which appear red; the second transmitted part of the middle-length waves (green); and the third, part of the short-waves (blue). Each projector had an independent voltage control. The observers picked an area, say a white one, and the experimenter measured separately the three (long-, middle- and short-wave) radiances coming from that area. Then the observer picked a second area, for example, a red one, and the experimenter measured the triplet of radiances coming from it. These measurements showed that there was slightly less long-wave light coming from the red paper than from the white, but that there was much less middle- and short-wavelength light. The experimenter then adjusted the amounts of the three illuminants so that the same triplet of long-, middle- and short-wave radiances came from the red paper as came previously from the white paper. For each waveband, the experimenter increased the illumination by the factor that the white paper was a better reflector than the red paper. All three illuminants were turned on together and the observer reported that the red area still looked red, even though the radiance measurements showed that the light reaching the eye was identical to that sent by the white area a moment before. The sensation red was produced by exactly the same stimulus at a point that previously produced the sensation white. In the same manner, Land went from

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[2] The experiment is so titled because the visual display used in the experiment resembles a painting by Piet Mondrian.

paper to paper in the display and produced very nearly the full gamut of color sensations with a single triplet of radiance measurements.

Land proposed that something fundamental was wrong with the idea that the biological system used the physical stimulus at a point to determine color. Instead of the long-, middle- and short-wave receptors comparing responses at a point, Land suggested that information from the long-wave receptors was intercompared to compute a biological analog of reflectance from the long-wave flux. Similarly, the information from the middle-wave receptors is intercompared to form the biological analog of reflectance for that waveband and this procedure is repeated again for the short-wave receptors. This biological analog of reflectance is called lightness. The information from each of the separate sets of cones generates a separate lightness image; the comparison of three separate lightnesses for each area is the determinant of color (Land, 1964).

The formation of the lightnesses and their comparison could occur in the retina or in the cortex. Experiments in visual physiology cannot as yet define the location of the interactions that must be occurring. Therefore, Land coined the word Retinex (made of "retina" and "cortex") to designate the physiological mechanisms that generate these independent images. His proposal did not demand that the retinal elements with the same sensitivity be directly connected to each other. Instead, somewhere in the retinal–cortical structure, elements with the same wavelength sensitivity cooperate to form independent lightness images (Land, 1964).

We wish to test whether the quantitative predictions of the Retinex theory match the color experience of an observer viewing the Color Mondrian experiments.[3] This test of Retinex theory is readily divisible into three parts. In Part I we quantify the color sensations seen by the observers. We asked our subjects to choose from the *Munsell Book of Color* the colored chips which best matched the color of each area in the Mondrian.

In Part II we test whether the observers' matches correlate with those predicted by Retinex theory. The theory states that each color is determined by a triplet of lightnesses, and that each lightness, in a situation like the Color Mondrian, corresponds to the reflectance of the area measured with a photodetector which has the same spectral sensitivity as one of the three cone pigments. Land's experiments show conclusively that color sensations do not correlate with the energy at each point. Our results show that the color sensations are very highly correlated with the triplets of reflectance. The results show that the visual system performs the analog of measuring reflectances even though it does not use known reflectance standards and invariant illumination.

In Part III we describe a model for calculating lightnesses from the radiances falling on each point on the retina. The calculations required by the model were performed by computer. The inputs to the computer were three arrays of radiances measured at 480 points on the Color Mondrian display. Each array was weighted by one of three absorption curves which characterize the cone pigments (Brown and Wald, 1963; Brown, unpublished). The output of the model was three arrays of computed lightnesses. We then test the model's predictions by comparing them with the triplets of lightnesses measured from the matching chips chosen by the observers. This comparison of observers' choices and computer calculations shows a very good fit.

PART I

To test the quantitative predictions of Retinex theory for the Mondrian experiment, we needed detailed psychophysical measurements of what the observer saw in each part of the experiment. In the past we have established standard black and white displays that can be used as a metric for lightness (McCann, Land and Tatnall, 1970). The principle was that an area in the standard display produced a constant, unique sensation, just as long as the illumination, the other areas in the display, and the state of adaptation of the observer's eye were all constant. A standard display satisfying these conditions can be used to quantify sensations produced in any other display having different illumination, surround or state of observer adaptation.

In the present experiments we are concerned with color. Our choice for the desired "catalogue" of color sensations was the *Munsell Book of Color* (Matte Finish Collection). We then had to select an appropriate illuminant. We could have used a broad-band illuminant such as Illuminant C, used in the original selection and definition of Munsell papers; instead we decided to use the three narrow-band illuminants used to illuminate the Mondrian. The reason for this choice is that the papers in the Munsell Book viewed in Illuminant C do not appear as saturated as the same Munsell papers viewed in three narrow-band illuminants. Rather than choosing papers for our Mondrian display which were less saturated than those used in the original experiment, we changed the convention of using Illuminant C on the Munsell Book. The uniform spacing of the elements in the Munsell Book which depend on Illuminant C was not essential for our use.

METHODS

The Munsell Standard

The Munsell Book was placed on the bottom of a large five-sided (75 cm) cubic box. The observer sat facing the open sixth side, resting his head in goggles suspended across the open side of the cube. The right side of the goggles was covered so that the subject could use only his left eye when viewing the Munsell Book Standard.

[3] We have reserved for later papers the comparison of Land's Retinex model with other explanations of the invariance of color sensation with changes in the wavelength-radiance distribution of the light coming to the eye, such as chromatic adaptation (von Kries, 1905). The literature contains many variations of the chromatic adaptation hypothesis (Helson, 1943), a few empirical formations such as that of Judd (1940), and a variety of experiments that articulate problems with theories which assert that chromatic adaptation can account for the absence of correlation between the wavelength-radiance distribution coming from a point and the color sensation of that point (Walters, 1942; Wassef, 1958, 1959; Land and Daw, 1962; Land, 1975).

Black velvet cloth, which surrounded the goggles, covered the remaining face of the cube. Three projectors, each with a narrow-band interference filter (peak transmission at 450, 530 and 630 nm; bandwidth 10 nm at half-height) sent light into the box through small ports in the wall opposite the observer. Mirrors reflected the three beams of light up onto the top inside surface of the box. The optics were such that each beam almost covered the top of the box, and the mirrors were placed so that all three beams were centered and roughly superimposed. The integrating properties of the box made it possible to illuminate each chip in the Munsell Book with the same quantity of each waveband, thus satisfying the requirement for even illumination. A 12·7 by 20·3 cm gray paper (Munsell Value 5/) was used as the constant surround. A hole slightly smaller than a Munsell chip was cut in the center of the paper. The observer moved the gray paper about and placed it around each chip he considered as a match. This satisfied the requirement that each chip have an identical surround. Finally, to keep adaptation effects as constant as possible, we conducted our experiments in a darkened room and used a binocular matching procedure. Both the experimental display and the Munsell Book were viewed monocularly but with different eyes. The subject had to turn away from the Mondrian display to look at the Munsell Book. Both displays were viewed through mounted goggles which automatically covered the eye used to view the other display. This successive comparison procedure eliminated any possible binocular interaction between target and standard (see Fig. 1).

We adjusted the voltages of the three projectors illuminating the Munsell Book so that the white area appeared the "best" white. The triplet of radiances coming from the white paper in the Munsell Book was $1{\cdot}15 \times 10^{-2}$ W. sr^{-1} m^{-2} 630-nm light, $7{\cdot}8 \times 10^{-3}$ W. sr^{-1} m^{-2} 530-nm light, and $3{\cdot}3 \times 10^{-3}$ W. sr^{-1} m^{-2} 450-nm light.

The Color Mondrian display

The Mondrian part of the experimental display was a 30-cm square piece of cardboard completely covered by 17 small rectangles of Munsell papers in various sizes and hues, surrounded by a uniform middle-gray area. This display, referred to as the 17-Area Mondrian (see Fig. 2), was viewed in a 90 × 60 × 60 cm illumination box with a white interior and a black exterior. The target was placed in the back of the illumination box and was held in place by a hinged door. The Mondrian and its background were illuminated by three projectors each with different narrow-band interference filters. The peak wavelengths of the three filters were 450, 530 and 630 nm. The flux coming from each projector could be independently varied by one of three variable transformers, each connected to a separate voltage-stabilized circuit. Mirrors were placed in the beams of light to divide each beam and spread the light uniformly inside the white box. No light from the projectors fell directly on the Mondrian display. All of the light falling on the target was reflected from the walls of the box, making the effective light source large and the illumination approximately uniform; the maximum deviation from uniform illumination for any one waveband was less than 10%.

To be sure that the subject had no way of estimating the illuminant, he looked through a mask, fitted over a hole in the wall opposite the Mondrian, which allowed him to see only the target and none of the interior white walls. His head was held in position by a lensless goggle mounted 104 cm in front of the Mondrian display. The sides of the goggle were opaque and thus prevented any light from other equipment from reaching the eye.

Procedure

For our initial measurements, the Mondrian illumination was identical to that falling on the Munsell Book. We measured the triplet of radiances coming from a gray paper (Area *P*) in the display. In 630-nm light alone there were $5{\cdot}8 \times 10^{-3}$ W. sr^{-1} m^{-2}, which we define as the radiance X; in 530-nm light there were $3{\cdot}2 \times 10^{-3}$ W. sr^{-1} m^{-2}, which we define as Y; and in 450 nm there were $1{\cdot}6 \times 10^{-3}$ W. sr^{-1} m^{-2}, which we call Z. The "gray experiment" is defined as 17-Area Mondrian with the illumination such that the triplet X,Y,Z radiances came from the gray area *P*. Then we chose a red-purple paper (Area *G*) and separately adjusted the three illuminants until the 630-nm radiance from the red-purple paper equalled X, the 530-nm radiance equalled Y, and 450-nm radiance equalled Z. This is defined as the "red experiment". Similarly, a blue paper (area *H*), a green paper (area *R*), and a yellow paper (area *C*) were also chosen. When the illumination was such that X,Y and Z came from these papers, that defined the "blue", "green" and "yellow" experiment. In each "experiment" there is one area in the Mondrian that sends to the eye the X, Y and Z radiances. These color papers were chosen to have about the same Munsell Value as the gray paper and, when possible, have the second highest chroma available in the matte surface Munsell Book. In order to allow our observers the opportunity to choose from the Munsell Book a higher chroma as well as a lower chroma, we did not choose the chips with the highest chroma. Neither the initial lighting nor the choice of colors is critical.

The illumination on the Munsell Book remained constant and was equal to the illumination used on the Mondrian in the "gray experiment". The subjects were given unlimited time to choose the Munsell colors which matched the 17 areas in the display and the gray surrounding the target. The subjects were not told which illuminants were lighting the target for any particular experimental session. We randomized the order in which the experimental conditions were presented. Three subjects participated in five experimental sessions for each of the five conditions. The subjects were a female and two males, all with normal color vision and very good color discrimination (based on their low scores from the Farnsworth–Munsell 100-Hue Test).

RESULTS

The first column in Table 1 lists the Munsell designation of the actual papers used in the 17-Area Mondrian. The second through sixth columns list the Munsell designation of the average of the Munsell chips chosen by the three subjects. Each of these columns (2 through 6) reports the results for one of five experiments. Each row reports the results for a single area in the 17-Area Mondrian display for all five experiments. Each Munsell designation is the average of 15 observer responses—5 trials for each of three observers. The average was computed by averaging hue, value and chroma separately and then finding the nearest Munsell paper. Although the subjects' choices are all close to the actual Munsell designation, their choices for the "gray experiment" are most similar because the illumination for the "gray experiment" was identical to the Standard Illuminant used for the Munsell Book. Recall that in each experimental condition, the illumination was changed so that the triplet of radiances coming from the gray area in the "gray experiment" was identical to the triplet of radiances coming from the yellow area in the yellow experiment, etc. Since the triplets of radiances were identical for these five areas, they are indistinguishable from each other by any means of color photometry. If we were to calculate the position on

Table 1. This lists the actual papers that made up each area in the Mondrian and the average chip chosen by the observer to match that area in each of the five experiments described in the text. The matching chips for the gray area in the gray experiment, the red area in the red experiment, the blue area in the blue experiment, the green area in the green experiment and the yellow area in the yellow experiment are enclosed in rectangular boxes. These areas, as described in the text, sent to the eye exactly the same wavelength-radiance distribution. Despite this fact the observer chose a variety of different papers and, hence, a variety of different wavelength-radiance distributions to match one and the same wavelength-radiance distribution.

	PAPER IN MONDRIAN	MATCHING CHIP IN MUNSELL BOOK				
Area		Gray Experiment	Red Experiment	Blue Experiment	Green Experiment	Yellow Experiment
A	N 5.75/	10.0 YR 6/1	7.5 G 6/2	5.0 YR 6/1	5.0 RP 5/2	10.0 PB 5/4
B	5.0 YR 5/6	5.0 YR 5/8	2.5 Y 5/4	5.0 YR 6/10	2.5 YR 5/8	5.0 YR 4/6
C	5.0 Y 8.5/10	5.0 Y 8.5/12	10.0 Y 9/10	2.5 Y 8/12	10.0 YR 8/10	**5.0 Y 8/8**
D	2.5 BG 6/6	5.0 BG 6/6	7.5 G 7/4	7.5 G 6/4	5.0 B 6/4	7.5 B 6/6
E	7.5 GY 6/6	5.0 GY 6/6	7.5 GY 7/8	2.5 GY 6/6	2.5 GY 6/4	10.0 GY 5/4
F	5.0 R 5/12	7.5 R 5/12	7.5 R 5/8	7.5 R 5/12	7.5 R 5/12	7.5 R 4/12
G	10.0 RP 6/10	2.5 R 6/10	**5.0 R 6/6**	5.0 R 6/10	2.5 R 6/12	5.0 RP 6/10
H	2.5 PB 6/8	2.5 PB 6/8	10.0 B 7/6	**2.5 PB 6/4**	7.5 PB 5/10	7.5 PB 5/12
I	5.0 GY 8.5/8	2.5 GY 8/10	5.0 GY 8/10	10.0 Y 8.5/10	7.5 Y 8/10	5.0 GY 8/6
J	10.0 RP 3/6	5.0 R 4/6	10.0 R 4/4	5.0 R 4/8	5.0 R 4/8	5.0 RP 3/6
K	N 9.6/	10.0 R 9/2	5.0 G 9/1	2.5 YR 9/2	7.5 RP 8/4	2.5 P 8/4
L	N 1.25/	N 1.50/	N 1.75/	N 1.75/	N 1.75/	N 1.75/
M	2.5 YR 7/10	2.5 YR 7/10	10.0 YR 7/6	2.5 YR 6/12	2.5 YR 7/12	2.5 YR 5/10
N	5.0 Y 7/8	2.5 Y 7/8	2.5 GY 7/8	2.5 Y 7/10	10.0 YR 7/10	10.0 YR 7/8
O	2.5 B 8/4	7.5 B 8/4	7.5 BG 8/4	2.5 B 8/2	5.0 PB 8/4	5.0 PB 7/6
P	N 6.75/	**5.0 YR 6/1**	10.0 G 7/1	5.0 YR 6/1	7.5 RP 6/2	10.0 PB 6/4
Q	5.0 P 7/6	10.0 P 7/6	7.5 PB 8/4	5.0 RP 8/6	10.0 P 7/8	2.5 P 7/8
R	2.5 G 7/6	10.0 GY 7/6	10.0 GY 8/6	7.5 GY 8/6	**10.0 GY 7/4**	2.5 BG 7/4

a chromaticity diagram of the identical triplets of radiances from these five areas they must, by definition, fall on exactly the same place on the graph; they have identical chromaticities and identical tristimulus values.

The five chips chosen by the subjects to match these five areas are: 5YR6/1, 5R6/6, 2·5PB6/4, 10GY7/4, 5Y8/8. These values are of particular interest because they indicate the range of colors that was achieved in this experiment by the same wavelength-radiance distribution. The chromaticities of the matching papers chosen from the Munsell Book for each of these five areas are plotted in Fig. 3. We have also indicated the total range of chromaticities available from the Munsell Book in the narrow-band illumination falling on it. The conclusion is clear. Very nearly the entire range of both chromaticities and color sensations that could be generated by the Munsell Book were generated by a single triplet of radiance measurements. As Land has shown, the wavelength-radiance distribution at a point cannot be the determinant of color sensations in the Color Mondrian experiments.

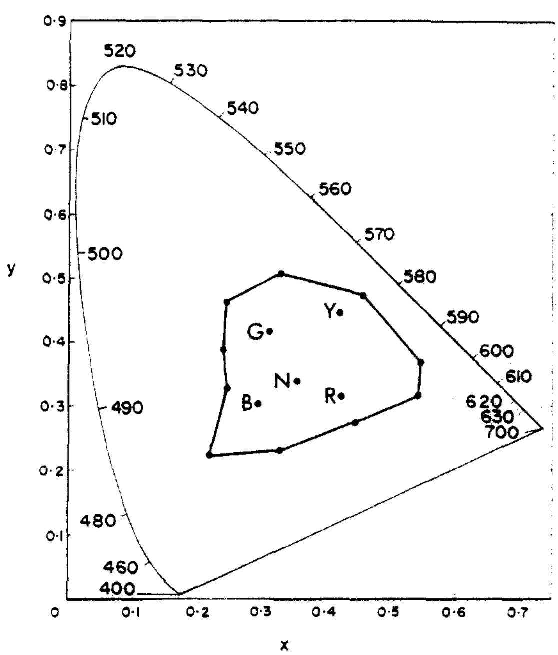

Fig. 3. The labelled points show the chromaticity coordinates of the Munsell chips chosen to match the five areas which sent identical wavelength-radiance distributions to the subjects' eyes. N labels the gray area in the "gray experiment", R the red area in the "red experiment", Y the yellow area in the "yellow experiment", G the green area in the "green experiment", and B the blue area in the "blue experiment". The unlabelled points connected by solid lines show the chromaticity coordinates of the most saturated Munsell chips on ten equally spaced pages (5·0 Hue) in the Munsell Book of Color when illuminated in our standard narrow-band illumination.

PART II

A test of the reflectance hypothesis

In this section we test the hypothesis that color sensations are determined by triplets of lightnesses, and that these lightnesses correspond to integrated reflectances in the present experiments. Note that we are using the term integrated reflectance, which is somewhat different from the usual physical definition of reflectance. Reflectance is the ratio of the radiance

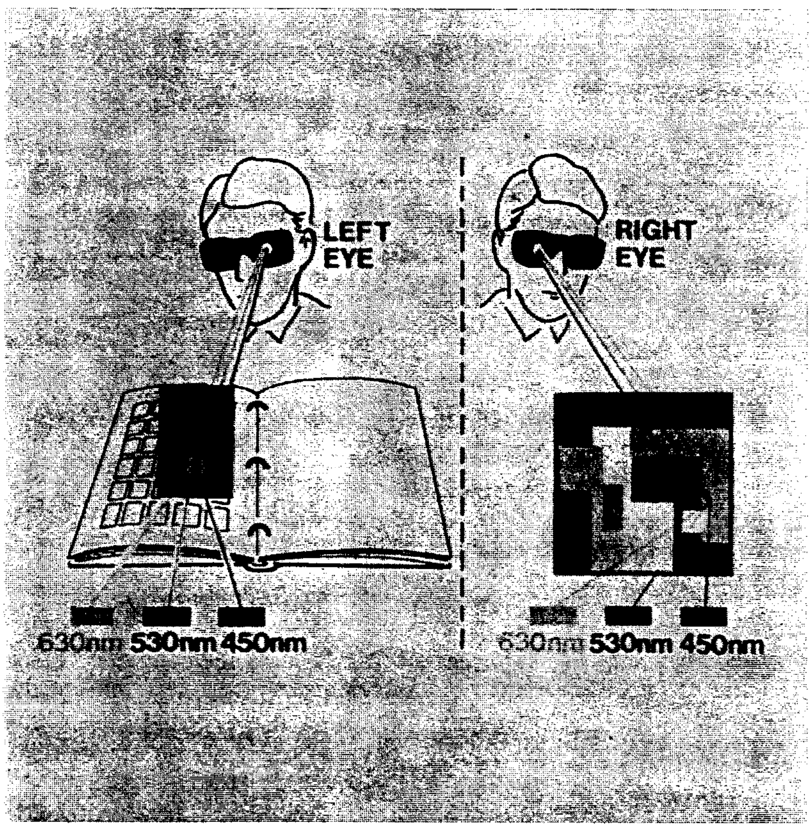

Fig. 1. A schematic diagram of the method by which the observers selected chips in the Munsell Book of Color which matched the areas in the 17-Area Mondrian. The observer views the Munsell Book with his left eye under fixed conditions namely, the illumination is constant and a gray surround is placed over each area. The observer views the Mondrian with his right eye. The illumination on the Mondrian is varied in each of the five experiments and the areas surrounding any particular area are arbitrary.

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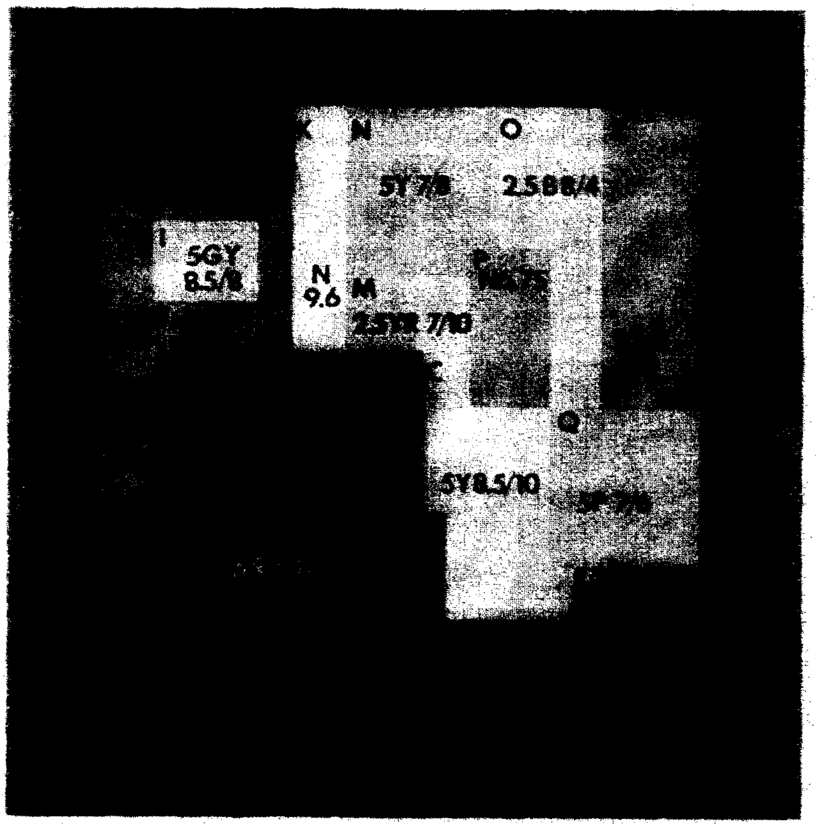

Fig. 2. A black and white photograph of the color display called the 17-Area Mondrian. The Munsell designation of each color area is printed on the photograph. Each area is also identified by a capital letter for future reference in this paper.

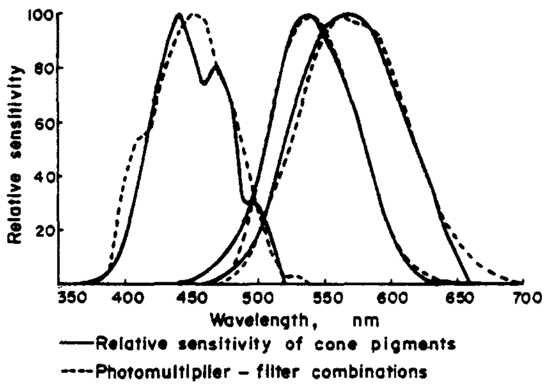

Fig. 4. The solid curves show the relative spectral sensitivity of human retinal cones as measured by Paul Brown, multiplied by the spectral transmission of the ocular media and the macular pigment. The dotted curves are the relative spectral sensitivity of the photomultiplier–filter combinations used to approximate the pigments. All curves are normalized to their peaks at 100.

reflected from a test object to the amount of light falling on the test object. Usually reflectance is measured by taking the ratio of the radiance from the test object to the radiance coming from a known reflection standard which reflects the same fraction of the light falling on it across the visible spectrum. Ordinarily, this pair of measurements is made by integrating radiance over a very narrow band of wavelengths, so as to specify the reflectance of the object at each wavelength. Instead, we are using three light detectors that each measure the reflected radiance integrated over nearly the entire visible spectrum. Each of the three integrated reflectance measurements is made with a light detector whose spectral response matches the spectral response of one of the three cone pigments. With each light detector we define the integrated reflectance of a paper as the ratio of the integrated radiance from that paper to the integrated radiance coming from a white standard paper. Thus, any colored surface can be characterized by a triplet of integrated reflectances weighted by the three spectral sensitivities of the cone pigments. This section of the paper tests how well this triplet of integrated reflectance measurements corresponds to the color seen by the observers.

METHODS

Many techniques have been used to measure cone-pigment sensitivity curves. The curves we used were Paul Brown's measurements (unpublished) of extrafoveal cones, obtained by the technique described by Brown and Wald (1963). These long-, middle-, and short-wave absorption curves were multiplied by the transmission of the eye and the absorption of the macular pigment to estimate the resultant net sensitivity of the pigments of the intact eye (Wyszecki and Stiles, 1967). We calculated the filter combinations that would alter the spectral sensitivity of the S-11 Photomultiplier surface in our Gamma-Scientific telephotometer to approximate most closely the corrected spectral sensitivity of each cone pigment. The results are shown in Fig. 4. The best fit for the long-wave pigment was provided by the combination of the photomultiplier and Wratten 106 and 8 filters. The middle-wave pigment was matched by the photomultiplier tube and Wratten 102 and 8 filters, and the short-wave pigment by the combined Wratten 47 and 86A filters.

We found the three integrated reflectances for every area in the Mondrian target using the following procedure. One of the experimental illuminants (all three projectors) described in the Methods section of Part I was set up. The radiance integrated under Brown's long-wave curve using the appropriate photometer–filter combination was measured from an area and recorded. This quantity is used as the numerator of the integrated reflectance fraction and is equal to the maximum integrated radiance that the long-wave cones could absorb from that paper. A large white paper used as a reflectance standard was placed in front of the Mondrian, and the integrated radiance under Brown's long-wave curve coming from it was measured. The ratio of radiances (paper/standard) was used as the long-wave integrated reflectance of the paper. This procedure was repeated for the middle-length wave and short-wave reflectances. We repeated these measurements for every area in the 17-Area Mondrian. Then, in turn, the four remaining experimental illuminants were set up and the reflectance measurements were made for each illuminant.

We also measured the integrated reflectances of each chip in the Munsell Book in our standard narrow-band illuminant. Thus, when our subjects chose a chip from the Munsell Book as a match for an area in the target, we could compare the three integrated reflectances of the target area with the three integrated reflectances of the matching chips.

RESULTS

Figure 5 is a graph of the radiances integrated under Brown's curves coming from areas in the Mondrian vs the radiances coming from the Munsell chips chosen to match them. On the graph we have plotted the results from all five "experiments" for each of 17 color areas and the gray surround for each of three wavebands. The solid line at 45° describes the locus of points where the radiance from the matching Munsell chip was equal to the radiance from the Mondrian area. There is very little correlation between integrated radiance from an area and its matching color sensation. For example, the Mondrian areas that sent to the eye 40 radiance units were matched by the Munsell Book areas that sent to the eye radiances varying from 15 to 70 units.

If, instead of radiances, we compare the triplets of integrated reflectances, we find good correlation between the measured properties of the 17-Area Mondrian and chips chosen from the Munsell Book. Figure 6 is a graph of integrated reflectance of the same Mondrian areas vs the integrated reflectance of matching Munsell chips. Just as in Fig. 5, the graph contains all five "experiments". The solid line describes the locus of points where integrated reflectances of Mondrian areas and matching chips are equal. Compared to radiance, integrated reflectance is a much better predictor of color sensation.

Before we can evaluate how well integrated reflectance predicts color, we must consider a problem in scaling. Land's hypothesis is that a triplet of lightnesses determines color, but what is the relationship between reflectance and lightness? Equal increments in reflectance do not represent equal increments in sensation. For example, the difference in lightness

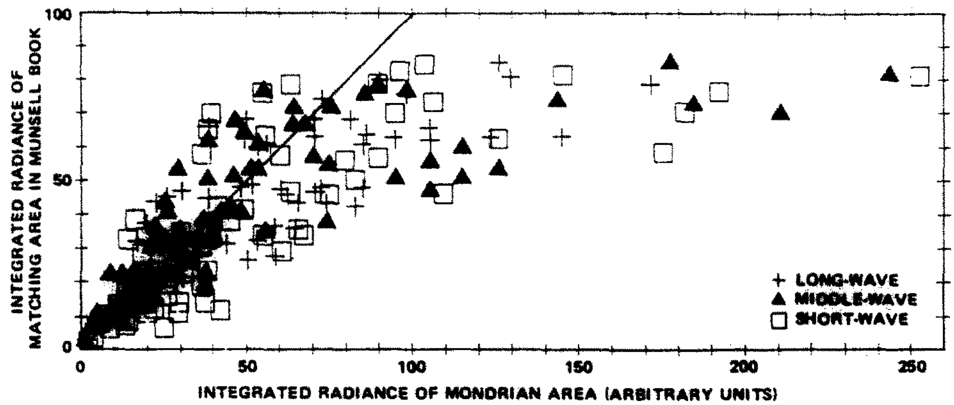

Fig. 5. Compares the integrated radiances coming from each Mondrian area (horizontal axis) with the average integrated radiances coming from the matching Munsell chips (vertical axis). All long-wave integrated radiances are plotted with the symbol +, all middle-wave radiances with ▲, and all short-wave radiances with ■. The data from all 17 areas in all five experiments are plotted here.

between two papers that have reflectances of 90 and 80% is very small while the difference in lightness between papers that have reflectance 15 and 5% is very large. The Munsell Value Scale from white to black was partitioned so that each increment in Value is a uniform increment in lightness (Newhall *et al.*, 1943). In order to be able to compare the significance of the differences between integrated reflectance, both measured from the Mondrian and chosen by the observers, we scaled all the reflectance measurements just as we had done earlier with Black and White Mondrians. Each increment of the scaled reflectance represented a constant increment of sensation. We converted each integrated reflectance to scaled integrated reflectance by using the Glasser *et al.* (1958) approximation to Munsell Value:

$$V = 2{\cdot}539\,\rho^{1\cdot 3} - 1{\cdot}838 \text{ for } \rho > 0{\cdot}384\%$$

where V is scaled integrated reflectance and ρ is per cent integrated reflectance. This scaled integrated reflectance goes from a value of 10·0 for a perfect reflector to 0·0 for the ultimate black. Neither of these lightnesses is attainable. We used a Color-Aid white paper as our white standard; it had an absolute reflectance of 93·5% and a scaled integrated reflectance of 9·6. In order to have the reflectance measurements conform to Glasser's equation, we multiplied each reflectance by 0·935.

Figure 7 is a graph of the scaled integrated reflectances of Mondrian areas and matching Munsell chips. At the 45° line, the scaled integrated reflectance of the Mondrian area equals the matching Munsell chip; the fit to the line is quite good.

Table 2 lists the scaled integrated reflectances of the Mondrian areas and the matching chips. The table lists two triplets of integrated reflectances for each area in each experiment. The left triplet lists the scaled integrated reflectances of the Mondrian areas

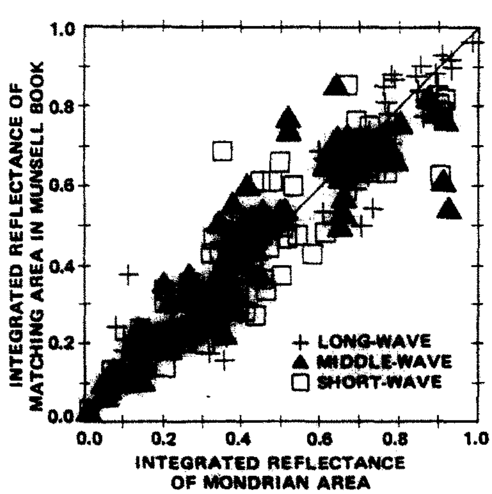

Fig. 6. The integrated reflectance of each area in the Mondrian (horizontal axis) is plotted against the integrated reflectance of the matching Munsell chip (vertical axis). All long-wave integrated reflectances are plotted with the symbol +, middle-wave with ▲ and short-wave with □.

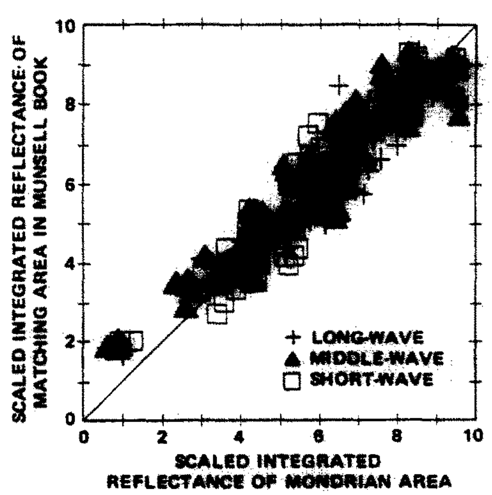

Fig. 7. The scaled integrated reflectance of each area in the Mondrian (horizontal axis) is plotted against the average scaled integrated reflectance of the matching Munsell chip (vertical axis). All long-wave scaled integrated reflectances are plotted with the symbol +, middle-wave with ▲ and short-wave with □.

Table 2. This lists the scaled integrated reflectances of each area in each of the five experiments. For example. the results from Area A in the gray experiment can be expressed as a set of six scaled integrated reflectances. Three of those values on the left are the long-, middle-, and short-wave scaled integrated reflectances of the paper in the Mondrian; while the three on the right are the long-, middle-, and short-wave scaled integrated reflectances of the matching chip in the Munsell Book

SCALED INTEGRATED REFLECTANCES

MA = Mondrian Area

MMC = Matching Munsell Chip

Area		Gray Experiment MA	Gray Experiment MMC	Red Experiment MA	Red Experiment MMC	Blue Experiment MA	Blue Experiment MMC	Green Experiment MA	Green Experiment MMC	Yellow Experiment MA	Yellow Experiment MMC
A	LONG	5.2	6.1	5.3	6.1	5.1	6.3	5.0	5.0	5.2	5.3
	MIDDLE	5.3	6.1	5.1	6.4	5.2	6.4	5.2	4.9	5.2	5.2
	SHORT	5.4	6.0	5.4	6.1	5.5	6.2	5.4	5.3	5.5	6.3
B	LONG	5.3	5.2	4.6	5.2	5.5	5.8	5.6	5.2	5.1	4.2
	MIDDLE	4.3	4.3	4.3	5.0	4.5	4.8	4.5	4.0	4.4	3.5
	SHORT	3.3	2.8	3.5	3.6	3.4	2.9	3.3	3.0	3.4	2.7
C	LONG	8.7	8.8	8.5	9.3	8.8	8.4	8.8	8.3	3.6	8.4
	MIDDLE	8.3	8.7	9.3	9.3	8.4	7.9	8.4	7.7	8.4	8.2
	SHORT	4.3	3.4	4.5	4.5	4.4	3.6	4.2	5.3	4.1	4.8
D	LONG	6.2	6.1	6.8	7.1	5.6	6.1	5.5	6.1	6.2	6.2
	MIDDLE	6.8	6.9	7.0	7.5	6.8	6.7	6.6	6.4	6.9	6.5
	SHORT	6.0	6.4	6.1	6.9	6.1	5.9	5.9	6.9	6.0	7.5
E	LONG	6.1	6.1	6.7	7.3	5.9	6.3	5.7	6.3	5.2	6.2
	MIDDLE	6.6	6.3	6.7	7.8	6.7	6.3	6.7	6.4	6.8	6.5
	SHORT	4.2	4.0	4.4	4.6	4.4	3.7	4.2	4.7	4.2	5.3
F	LONG	6.5	5.8	4.3	5.4	6.9	5.8	7.1	5.7	5.3	5.3
	MIDDLE	3.6	3.8	3.1	4.2	4.1	3.7	4.3	3.7	3.5	3.6
	SHORT	3.6	3.0	3.7	4.0	3.7	3.0	3.6	3.0	3.7	3.8
G	LONG	6.8	6.7	5.2	6.1	7.2	6.5	7.5	6.8	6.5	6.7
	MIDDLE	4.6	5.3	4.3	5.4	5.0	4.8	5.1	5.1	4.5	5.3
	SHORT	5.3	5.4	5.3	5.6	5.4	4.9	5.2	5.3	5.4	6.4
H	LONG	5.9	6.1	6.4	7.1	5.8	6.4	5.7	5.1	6.1	5.0
	MIDDLE	6.4	6.5	6.4	7.5	6.5	6.7	6.5	5.3	6.5	5.2
	SHORT	7.8	8.2	8.0	8.6	8.1	7.9	7.9	7.8	8.2	7.9
I	LONG	8.6	8.5	9.0	8.2	8.6	8.9	8.6	8.5	8.7	8.2
	MIDDLE	8.9	8.7	8.9	8.6	9.1	8.9	9.0	8.4	9.0	8.5
	SHORT	5.2	4.0	5.4	4.2	5.5	4.3	5.1	4.1	5.2	5.9
J	LONG	3.9	4.4	3.0	4.1	4.3	4.5	4.3	4.5	3.8	3.5
	MIDDLE	2.7	3.5	2.4	3.5	2.8	3.3	2.9	3.3	2.7	2.8
	SHORT	3.1	3.5	3.1	3.4	3.2	3.3	3.0	3.3	3.0	3.6
K	LONG	9.5	9.1	9.6	9.0	9.6	9.1	9.6	8.2	9.6	8.2
	MIDDLE	9.5	9.0	9.4	9.2	9.6	8.9	9.6	7.8	9.6	8.1
	SHORT	9.5	9.0	9.5	9.1	9.6	8.9	9.5	8.2	9.5	9.1
L	LONG	1.0	1.7	1.1	1.8	0.8	1.9	0.7	1.8	1.0	1.9
	MIDDLE	0.9	1.8	0.6	1.9	0.8	1.9	0.8	1.8	0.9	2.0
	SHORT	0.9	1.8	1.3	2.0	1.0	2.0	0.9	2.0	1.0	2.0
M	LONG	7.6	7.4	6.3	7.2	8.0	7.0	8.2	7.4	7.3	6.3
	MIDDLE	6.0	6.1	5.3	6.7	6.1	5.6	6.2	6.1	5.9	5.4
	SHORT	4.5	4.7	4.5	5.0	4.5	4.0	4.4	4.6	4.4	3.8
N	LONG	7.4	7.4	7.2	7.4	7.6	7.4	7.6	7.3	7.4	7.3
	MIDDLE	7.2	6.9	7.2	7.7	7.2	6.9	7.2	6.7	7.2	6.8
	SHORT	3.8	4.1	3.8	3.9	3.9	3.3	3.7	3.4	3.7	4.3
O	LONG	7.7	8.1	8.4	8.2	7.5	8.2	7.4	8.1	7.8	7.3
	MIDDLE	8.5	8.4	8.4	8.7	8.3	8.5	8.2	8.3	8.4	7.5
	SHORT	8.3	9.0	8.4	8.7	8.4	8.6	8.5	9.2	8.5	8.9
P	LONG	6.7	6.2	6.8	7.1	6.7	6.2	6.7	8.1	6.7	6.1
	MIDDLE	6.7	6.2	5.7	7.3	6.7	6.1	6.7	5.9	6.7	6.1
	SHORT	6.6	6.1	6.8	7.2	6.8	6.1	6.8	6.3	6.8	7.2
Q	LONG	7.4	7.4	7.2	8.1	7.5	8.4	7.6	7.4	7.3	7.1
	MIDDLE	7.0	6.9	7.0	8.1	7.1	7.6	7.2	6.8	7.1	7.0
	SHORT	8.1	7.9	8.3	8.3	8.3	8.3	8.3	7.9	8.3	8.6
R	LONG	6.7	7.2	7.5	8.2	6.5	8.5	6.3	7.1	6.8	7.1
	MIDDLE	7.6	7.7	7.6	8.8	7.6	8.9	7.4	7.5	7.5	7.6
	SHORT	5.9	5.7	5.3	5.7	5.8	6.7	5.7	6.1	5.7	7.2

measured under the illumination conditions in which the observer made the match. The right triplet lists the scaled integrated reflectances of the chip that the observer chose. As in Table 1 each column lists an experiment and each row lists an area in the 17-Area Mondrian.

Although scaled reflectance is a very good predictor of color sensations, careful scrutiny of Table 2 will show small differences in scaled reflectances for some areas and systematic shifts from one experiment to another. We computed the difference between the scaled integrated reflectances measured from the matching chip and those measured from the actual target area. The average difference was $+0{\cdot}1 \pm 0{\cdot}7$ (1 S.D.) on a scale of 0–10. Generally, the scaled integrated reflectances measured from the target area was very close to the scaled integrated reflectances of the chip chosen by the observer. but there were quite significant differences for some areas. There are two major reasons why some data points do not fall exactly on the 45° line. First, random errors associated with the psychophysical measurements of observer response may affect the results. Secondly, there is a systematic departure from a perfect correlation between color sensations and scaled integrated reflectance.

In the random error category, it is possible that there are errors introduced by inaccuracies in the shape of the cone pigment absorption curves or the lack of a perfect fit to these curves for the telephotometer. However, the effect of these errors is small because we measured the integrated reflectance of both Mondrian and Munsell Book with the same telephotometer and, hence, errors in spectral sensitivity tend to cancel. Comparison of matches by one observer with those of different observers also show that observer variability is very small.

A more significant source of the discrepancies between the scaled integrated reflectances of the Mondrian areas and those of the matching chips is the limited nature of the comparison standard. In some instances the Munsell Book may not contain papers whose reflectances closely resemble the integrated reflectances of areas in the Mondrian in nonstandard illumination. The observer may see the hue he wants, but not the value or chroma; the resulting compromise choice necessarily introduces errors.

This problem is of particular importance when we note that the integrated reflectance of Mondrian areas changes slightly with changes in illumination. It is the distinction between reflectance at each discrete wavelength and integrated reflectance that explains an object's ability to change integrated reflectance with change in illumination. Of course, an object's reflectance at any wavelength is by definition independent of illumination. However, integrated reflectance by its definition is subject to change. The integrated reflectance (ρ_c) for each different visual pigment c can be expressed as

$$\rho_c = \frac{k \sum_{\lambda=630,530,450} [R_p(\lambda).H(\lambda).V_c(\lambda)]}{k \sum_{\lambda=630,530,450} [R_s(\lambda).H(\lambda).V_c(\lambda)]}$$

where the numerator is the radiance sent to the eye from the paper p, and the denominator is the radiance sent from the known standard s. R is reflectance; H is irradiance; V_c is the spectral sensitivity of the visual pigment c; λ refers to one of three narrow-band illuminants at 630, 530 and 450 nm; k refers to the conversion factor from irradiance to radiance. Changing the illumination falling on the area changes only the three values $H(\lambda)$. Nevertheless. the integrated reflectances of a non-neutral color $R_p(\lambda)$ can vary considerably with λ. The product of $V_c(\lambda)$ and $H(\lambda)$ determines the relative contribution of $R_p(\lambda)$ to the sum. It is this change in weighting that changes p_c with changes in $H(\lambda)$.

Glancing across the row of Mondrian scaled integrated reflectances in Table 2 for Area P, we see very little change with the changes of illumination, a result that holds for long-, middle- and short-wave integrated reflectances. This is to be expected because the gray paper has nearly the same reflectance for all wavelengths. The green paper—Area R— exhibits more typical changes in integrated reflectance due to changes in illumination. The long-wave scaled integrated reflectance varies from 6·3 to 7·5. The middle-wave integrated reflectances varies less, from 7·4 to

7·6, and the short-wave reflectance also shows a small change from 5·7 to 5·8. The red paper, Area G, shows the maximum change: 5·2 to 7·5 in long-wave, 4·3 to 5·1 in middle-wave and 5·3 to 5·2 in short-wave light. These numbers also illustrate that the long-wave scaled integrated reflectances are most affected by illumination changes.

Area *G* is also a good example of an error introduced by the finite nature of the Standard and the change in scaled integrated reflectance with change in illumination. In the "green experiment", Area *G* had scaled integrated reflectances of 7·5, 5·1 and 5·2; the average scaled integrated reflectances chosen by the observers were 6·6, 5·1 and 5·3. The middle- and short-wave reflectances of the chip chosen by the observer match quite well with the measured Mondrian reflectances. The observers' choice of 6·6 for the long-wave was much lower than the 7·5 measured from the Mondrian, indicating much greater saturation of the area. The observers' choice specified in Munsell coordinates is 2·5R6/12. The matte surface Munsell Book does not contain a 2·5R6/14 or 2·5R6/16, one of which would be necessary to arrive at the reflectances measured from that Mondrian area in the "green experiment". The limitations of the finite Munsell Book are most apparent with white and black papers (Area *K* and Area *L*). All the Munsell chips above a lightness of 9·0 and below a lightness of 2·5 have a chroma of zero. If the observer wanted to pick an area of very high or very low value but with non-zero chroma, he would have to choose a darker or lighter chip to specify the hue. This explains why Areas *K* and *L* show comparatively large discrepancies between Munsell chips and Mondrian areas reflectances.

Let us now turn our attention to the small systematic differences found between various experiments. In order to produce the situation in which Area *G* in the "red experiment" sent to the observer's eye exactly the same radiation as had come from Area *P* in the "gray experiment", we had to increase all three illuminants. The irradiances were increased by factors of 1·91, 2·87 and 1·68. For the "red experiment", the average measured differences between scaled integrated reflectances of the matching Munsell chips and the scaled integrated reflectances of the Mondrian areas were +0·5 (long-wave), +0·8 (middle-wave) and +0·3 (short-wave reflectance). Compare this to the gray experiment: +0·1 for the long-wave, +0·1 for the middle-wave, and −0·0 for the short-wave integrated reflectances. It appears that increasing the illumination increases slightly the scaled integrated reflectances of the Munsell chips chosen to match the areas in the Mondrian. Using the data from all five experiments, we tested whether overall illumination affects the subjects' choices. Figure 8 is the graph of the ratio of overall illumination (radiance from Area *K*, a white paper, in any "experiment" to radiance from the same paper in the Munsell Book Standard) vs the average difference in scaled integrated reflectance. The 15 data points (plotted as ×'s) show that there is a systematic effect on the appearance of Mondrian areas due to overall changes in illumination. When the illumination is greater than that of the Standard, the observer matches the Mondrian area with a paper that has a slightly higher reflectance. When the illumination is less than that of the Standard, the observer chooses a lower reflectance chip. These small departures (less that 10% integrated reflectance or 0·8 scaled integrated reflectance) from perfect correlation between Mondrian areas and matching Munsell chips represent the most serious discrepancy in the hypothesis under examination.

As an additional test of the influence of illumination, we performed a control experiment in which we changed the intensity of all three wavebands by the same amount. In the "gray experiment" the illumination of the target was the same as that falling on the Munsell Book. We increased the intensity of the three wavelengths illuminating the target until each was double that falling on the Munsell Book; we also halved the three illuminants. Our subjects matched

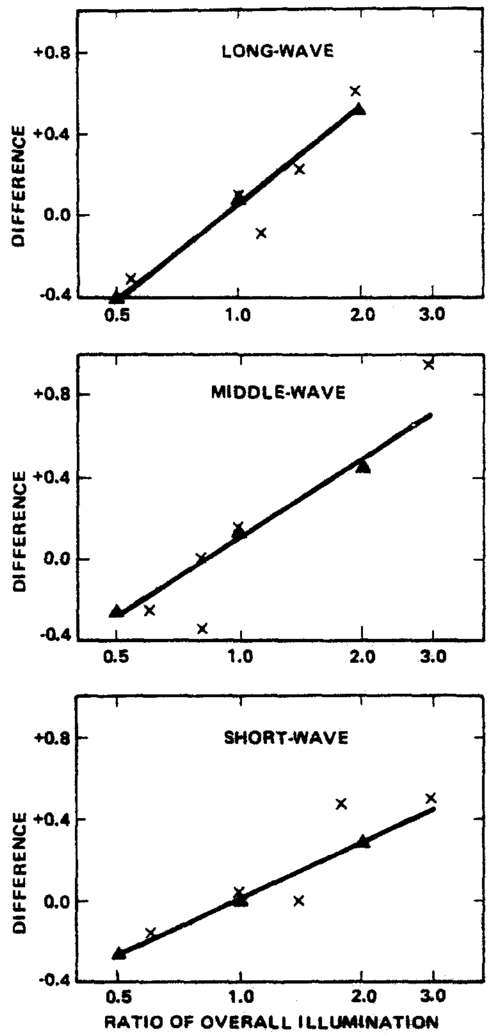

Fig. 8. These are graphs of differences in scaled integrated reflectance (matching chip in Munsell Book minus Mondrian area) as a function of overall brightness. The vertical axis plots the average difference in scaled integrated reflectance, while the horizontal axis plots the ratio of overall illumination (radiance coming from the white paper, Area *K*, in the Mondrian divided by the radiance coming from the white paper, N9·6/, in the Munsell Book). The +'s plot the data from the five color "experiments", while the filled triangles and solid line connecting them plot the data from the control experiment in which all three illuminations were changed by the same factor.

the target areas to the chips in the book at each of these three light levels: double, equal or half. In Fig. 8, the filled triangles, connected by the solid lines, show the results of this experiment. Observers selected chips with a slightly higher reflectance when the illumination was increased, and they chose chips with a slightly lower reflectance when the illumination was decreased. The systematic differences found in the five "experiments" (plotted as ×'s) appear to fall about the solid lines. Thus, we conclude the systematic differences found in Table 2 are due to differences in overall illumination between the Mondrian and the Munsell Book.

PART III

Correlation of observers' choices with model's predictions

The results of the last section have shown that the primary function of a mathematical model for these experiments is to calculate the integrated reflectances of each area in the Mondrian. The problem becomes more interesting when we remember that the eye does not have the usual tools of the physicist. The eye has neither a reflectance standard nor constant illumination. A model for color vision must propose a way of calculating three integrated reflectances from the wavelength-radiance distributions that fall on the retina.

The first of the five assumptions of our color model is that there are three different types of light transducers, i.e. receptors, each with a different spectral sensitivity described by one of Brown's cone pigment curves. This hypothesis is based on Thomas Young's (1802) suggestion and is held in common with almost all theories of color vision.

The second assumption is that the model compares all the information from all the long-wavelength receptors to determine the equivalent of long-wave reflectance for each area. As we will discuss later these interactions need not be direct interconnections between receptors. Information generated by the long-wavelength receptors is intercompared across the entire field of view. The model arrives at the integrated reflectance of each area by comparing the long-wave response for each area to the long-wave responses for all other areas. Thus, the highest reflectance in the entire image becomes the "reflectance standard". It is possible to calculate the long-wave reflectance from all the long-wave receptors' responses to energy only if the information received these receptors is kept separate. We simply apply this process three times, once for each type of light receptor. This procedure is repeated for the responses of the middle- and short-wavelength receptors. Each system acts independently to produce separate lightness values, which approximate integrated reflectances for each point in the visual image. This second assumption comes from Land's Retinex theory and is a significant departure from Young's theory, which proposes intercomparisons of long-, middle- and short-wave receptors at a point.

The third assumption of the model is that changes in reflectance are discontinuous, abrupt, and highly visible while changes in illumination are continuous, slow and nearly invisible. Therefore, gradual reflectance changes are regarded by the visual system as changes in illumination and hence are nearly invisible; while discontinuous changes in illumination, such as shadows with sharp boundaries, are regarded as changes in reflectance. If two points in the field of view are close together, the difference in illumination falling on these points will be small, even in displays in which the illumination varies considerably from one side to the other. In order that the ratio of radiances from two closely spaced points be significantly different from 1·0, these two radiances must be measured from two areas with different reflectances. Since the difference in illumination is small, the ratio of radiances of these two areas will be close to the ratio of their reflectances, and in fact the ratio of radiances approaches the ratio of reflectances as a limit when the distance between two points on either side of an edge approaches 0·0. It follows that a simple mechanism for finding the ratio of reflectances of two adjacent areas is a bridge pair of receptors that computes the ratio of radiances at closely spaced points.

The fourth assumption is that the model takes the reflectance ratios generated by the previous stage and sequentially multiplies them to form a product of the ratios at each point in the image. The ratio stage determines the relationship of any area to all of its adjacent areas, while the sequential product stage determines the relationship of any area to all other areas in the entire field of view. This multiplication can be done in a variety of ways. The simplest is to generate a series of paths that wander through the two-dimensional array of energies formed by the display on the model's "retina". Each path can begin anywhere in the image. The operation along each path takes the ratio of two adjacent points and multiplies it by the ratio of the next pair of points along the path. At any point in a given pathway through an array of radiances, the product equals the ratio of the reflectance of the last area in the sequence to the reflectance of the starting point. The name sequential product is meant to imply a sequence of operations in which each point in the field of view is able to influence other points, even distant points. We are not implying that the visual system computes its lightness response in the manner of a digital computer, i.e. one ratio and product at a time. The model as described in this paper is one of many embodiments using the ratio-multiplication process. It is the description of a digital, two-dimensional computation for testing the processing principles. It uses standard serial processing suitable for computer analysis. Details of parallel processing systems that might resemble biological components easily follow from the processing principles but are beyond the scope of this paper.

If each of the digital computer's paths began in a region of 100% reflectance, then the sequential product at each point would be numerically equal to the reflectance of that area. Instead, the model assumes that the first value in any sequence is a 100% reflectance, and the model sets any value of the sequential product greater than 1·0 equal to 1·0. This resetting procedure would occur whenever the path reaches an area whose reflectance is greater than the actual reflectance of the starting point: it ceases after the path reaches the highest reflectance in the scene, and thereafter each of the sequential products equals the ratio of reflectance of that area to the highest reflectance. The sequential product converts the relative reflectances of two adjacent areas to the reflectance compared to the highest reflectance in the field of view. The third and fourth assumptions of the model come from Land and McCann's model for lightness (see Land and McCann, 1971).

The fifth assumption is that the model must have the ability to arrive at lightness in situations in which lightness does not correlate with reflectance. There are two general cases in which the lightness of objects fails to correlate with reflectance. The first are situations in which two areas of identical reflectance appear different when placed in markedly different surrounds, usually called simultaneous contrast. (Since we used a multicolored test target in these experiments, the influence of surround is difficult to isolate.) Secondly, two areas of the same reflectance will also not match if the intensities of their illuminants are very different. Large changes in overall illuminance produce small changes in lightness. It is a common observation that objects seen on a bright sunny day have different lightnesses from the same objects seen on a dark rainy day. This fact has also been experimentally measured by Jameson and Hurvich (1961), Stevens and Stevens (1963), and Bartleson and Breneman (1967).

The departure of lightness from dependence on reflectance plays a small but important part in these experiments. In order to calculate lightness rather than just scaled integrated reflectance, there must be a small dependence of lightness on changes in the overall level of illumination. The data from Fig. 8 provide us with measurements of changes in lightness as a function of luminance for experiments described in this paper.

The assumption about overall illumination makes it possible to explain how a pure spectral band of wavelengths in a completely dark surround appears to have color. Without a small correction for large changes in overall radiance the model would predict that any single wavelength in the spectrum would appear white when presented in the absence of any other light. For example, consider a spot of 520-nm light. All three of Brown's pigment curves show a response to this wavelength. The long-wave pigment is 60% as sensitive to 520 nm as it is to 560 nm (peak of long-wave pigment). The middle-wave pigment is 90% as sensitive to 520 nm as it is to its peak (530 nm), and the short-wave pigment 1% as sensitive to 520 nm as it is to its peak (440 nm). If we were to process this target with the rules given above the model would report that the 520-nm spot was at the top of the lightness scale on all three Retinexes and therefore should be assigned 100% reflectance or a lightness of 9·6. When all three lightnesses are 9·6, the model predicts that the color is white; 520 nm light is not white, but green. However, there is 90 times more radiance available to the middle-wave cone pigment than to the short-wave pigment and three times more radiance available to the middle-wave pigment than to the long-wave pigment; changes of this magnitude must have an effect on the apparent lightness of the spot on each Retinex. We conclude that the short- and long-wave lightnesses of the 520-nm spot will be less than that of the middle-wave lightness. Green sensations are characterized by triplets of lightnesses in which the middle-wave lightness is greater than the other two.

Calculation of model predictions

The first step in the process was to measure the radiances of each point in the simplified Mondrian in the long, middle, and short wavebands. With all three narrow-band illuminators on simultaneously, the integrated radiances under each of Brown's receptor curves were measured. For each receptor sensitivity, the Mondrian was characterized by 480 radiances spaced regularly in a 24 × 20 array. The computer model was not given the positions of boundaries of areas, just the radiance at all points. We chose to use unidirectional paths of length 200 for all three model calculations. The origin of the path and its direction in the 480-point array were determined by a random-number generator. The paths traveled straight ahead until they reached the perimeter of the target where they either reflected back across the target or traveled along the perimeter. The direction of the reflection from the perimeter was also chosen by the random-number generator. At each point along the path the radiance from that point was divided by the radiance at the previous point. The ratio was tested to see whether the difference from 1·0 was significant. A ratio between 0·997 and 1·003 was set equal to 1·0; then the ratio was multiplied by the sequential product from the previous point. This sequential product was tested to see if it was larger than 1·0. If so, it was set equal to 1·0, initiating a new high reflectance standard. If not, it was sent on unchanged. This sequential product was used twice: first, it was held to be averaged with all the other outputs from paths that had reached this location in the array; and second, it was sent on to be multiplied into the next sequential product. The geometric mean of all the sequential products at each point was computed and this average sequential product used as the prediction of the model. Using a correction factor for overall illumination based on the solid lines in Fig. 8, we applied a normalizing factor to each sequential product calculation. With these corrections, the model went beyond the computation of integrated reflectance (a property that can be measured with a meter) to an estimation of lightness (measurable only by a visual system). Therefore, we will call the scaled output of the model "computed lightness". The observer quantified each sensation by matching it to the Munsell Book. We define the scaled integrated reflectances of Munsell chips in the Standard to be equal to the lightnesses chosen by the observer. Figure 9 has 15 graphs, each of which plots computed lightnesses on the vertical axis and observed lightnesses on the horizontal axis. There is one graph for each cone waveband in each "experiment". The left-hand column contains the long-wave graphs for each of the five experiments; the middle column, the middle-wave graphs; and the right hand column, the short-wave graphs. The "gray", "red", "blue", "green" and "yellow" experiments are presented in successive rows from top to bottom. The letters in

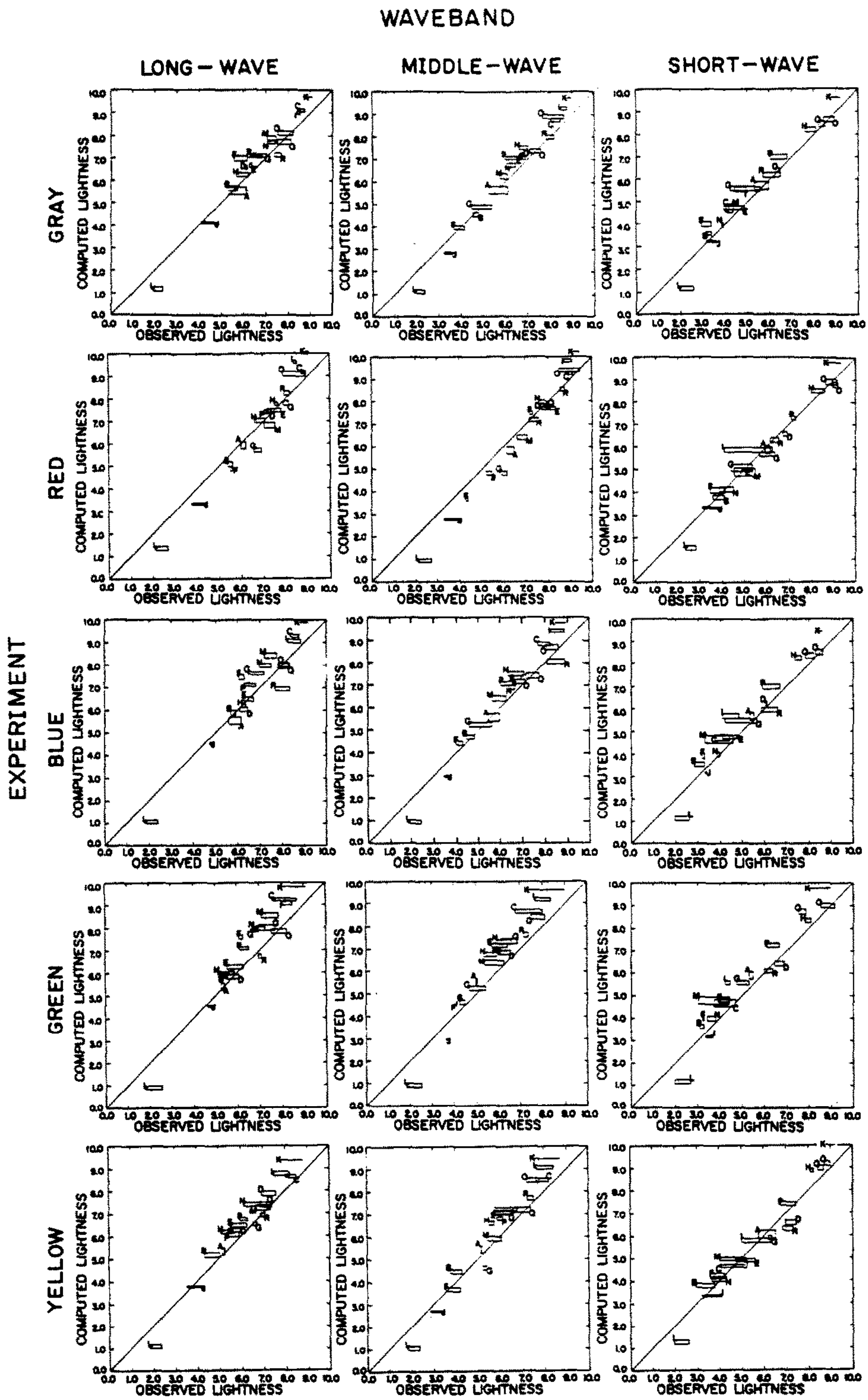

Fig. 9. Shows the long-, middle-, and short-wave observed lightness vs computed lightness comparisons for each of the five experiments. The height of the boxes is the mean computed lightness ± 1 S.D. of the various individual predictions for points in the array that make up that area. The width of the box is the mean ± 1 S.D. of the observers' choices of matching lightness.

each graph identify each area of the Mondrian. The center of each box is the mean of computed vs the mean of observed lightnesses. The predictions of the model are in excellent agreement with the chips chosen by the observer.

DISCUSSION

We have shown that the model can predict observer results with considerable success. The model is very simple. Its major function is to find a triplet of reflectances, one for each photopigment, without a reflectance standard. The model achieves this, first by processing the long-wave information independently of the middle- and short-wave information, and second by comparing each area to every other area within a given waveband. The particular technique of computation is one of many equivalent embodiments of the model. The fundamental assumptions are merely that the ratio of energies, at closely spaced pairs or sets of points, are multiplied to a give a product that is closely correlated with reflectance and represents long-distance interactions across the image.

How well does this model agree with the results of other psychophysical experiments? The Mondrian experiments show that information from each set of cones is processed independently. Other studies, using threshold measurements, show independence of cone mechanisms at stages of the visual process measured by that particular threshold. Stiles (1949) concluded that the cone mechanisms were independent. Alpern's (1965) metacontrast experiments showed that the threshold for a 5-msec flash can be greatly increased by following it with a second flash surrounding the first. Alpern and Rushton (1965) extended the desensitizing metacontrast experiments to test the independence of the cone mechanisms. They found that if a test flash excited one cone mechanism, then the after-flash raised that threshold only to the extent it stimulates that mechanism in the surround. With achromatic targets, Westheimer showed that illuminating the area surrounding a test flash either increased (desensitization) or decreased (sensitization) its threshold depending on whether the diameter of the region surrounding the test flash was small or large. McKee and Westheimer (1970) measured the action spectrum of the sensitizing effect for the red and green color systems. They showed that sensitization occurred primarily within a particular cone mechanism, not between cone mechanisms.

Despite considerable psychophysical evidence for independent processing, neurophysiological data from some stages in the visual pathway suggest non-independent processing similar to that proposed originally by Hering (1964). In the goldfish, Svaetichin (1956) and MacNichol *et al.* (1958) found units, later shown to be horizontal cells, which were depolarized by long-wavelength light and hyperpolarized by middle-wavelength light. Recordings made in monkey lateral geniculate cells (DeValois, 1965; Wiesel and Hubel, 1966) showed that the receptive field center responded to one spectral region, while the surround responded antagonistically to another spectral region.

The combination of these results indicate the information from one cone mechanism is combined with the information from other cones at certain neural stages. The interaction of information between different cone mechanisms observed in the neurophysiological work is not necessarily in contradiction to the psychophysical results. There are two distinct problems confronting the human color system. The first is calculating the reflectance of each area from the radiances from every point. The second is the maintenance and possible enhancement of the differences in spectral information obtained from the different photopigments. The overlap of the absorption curves of the visual pigments is so great that the maximum difference between the long-wave pigment response and the middle-wave pigment response is very small, even for brightly colored papers. For example, the integrated reflectances of the reddest chip in the matte surface Munsell Book (5R5/12) are 34, 15 and 11%. The integrated reflectances of the chip that is most different in color (5BG5/8) are 20, 27 and 23%. This very restricted range of differences in integrated reflectances is due to the extent of overlap of the cone pigments. Compare these differences of integrated reflectance with the difference between three 90% reflectances for a white and three 4% reflectances for a black. Opponent processing of the color signals at some stage could be invaluable for reliable transmission of spectral information to higher levels. Opponent processing may have nothing to do with the conversion of radiance at every point in the retina into sensations that correspond to the reflectances of areas in the field of view. It may be used to guarantee accurate transmission of spectral or color information from one location of lateral interactions—the retina—to another location of lateral interactions—the cortex. With a transmission system which contains a finite amount of noise, it seems more beneficial to transmit the small difference between two large numbers than to transmit both large numbers.

There is analogous appearance and later disappearance of opponent-type processing in the transmission of color television signals. The color television camera uses three vidicon tubes, each filtered to respond to one spectral region. The intensity of the light at each point in the image is determined separately for each waveband. In the color television set there are three electron guns, one for each set of red-, green- and blue-emitting color phosphors. However, the signals that are transmitted from the television station to the individual receivers are not three independent signals, but these signals are coded by a system somewhat like the opponent processing first suggested by Hering. The comparison of the visual system with television is good for illustrating how opponent processing of the signals is helpful for transmission of signals over long distances. The television analogy is poor in that television detects, transmits and reproduces an equivalent set of radiances on the face of a cathode ray tube. There are no color sensations actually produced by color television; one needs a visual system to generate color sensations.

This use of opponent processing leads to three equally good alternative hypotheses about the human visual system. The first is that the reflectance of each object in the field of view is established very early in the neural processing and that the signal transmitted to the cortex correlates with the reflectance of objects, not the radiance absorbed by photopigments.

If this hypothesis is correct, then the reflectance calculations proposed by Retinex theory have been completed before the signals are sent to the cortex and before opponent signal processing can be used to enhance color and transmission properties for the lightness signals.

The second hypothesis is that the reflectance calculations do not occur in the retina but are located in the cortex. This hypothesis can as well make use of spectral opponent processing for enhancement and transmission, but would require reconstruction in the cortex of three radiance arrays absorbed by each type of cone.

The third hypothesis is that part of the reflectance calculation takes place in the retina and part in the cortex. Here again opponent processing could be used for its transmission properties.

Choosing one of these three hypotheses is not possible until more is known about the quantitative properties of signals recorded from the intermediate cells between retina and brain. If reflectance is established in the retina, then the signals in the ganglion and lateral geniculate cells correspond to the lightnesses of objects and not to the radiances coming to the eye. If the second hypothesis is true, then the signal recorded from the ganglion cells and the lateral geniculate will correspond to the radiance coming to the eye and show little correlation to sensations reported by the observer. If the third hypothesis is true, namely, that the calculation of reflectance takes place partially in the retina and partially in the cortex, then the signals recorded from the ganglion and lateral geniculate cells would correspond to relative reflectance calculations for a limited field of view, whereas signals recorded from cells in the cortex would correspond to reflectance calculated over much greater angular subtends or the entire field of view.

Recent neurophysiological experiments seem to support the general scheme that color opponent processing is an intermediate step in the processing of color information. Fuortes and Simon (1974) recently reported that L-type horizontal cells in turtles received input from only one type of cone. They also reported that the color-opponent C-type horizontal cells did not receive signals from red cones, but from red horizontal cells responsive to a large receptive field. This kind of opposition involving the response over a large area is distinctly different from that of the opposition of a green cone or a group of green cones with a surrounding group of red cones in a comparatively small area. Although there are many similarities to be found in turtle and primate retinas, hypotheses must be tempered by the fact that there are as well many differences between these structures (Daw, 1973).

Neurophysiological details of primate ganglion, lateral geniculate, and cortical cells are available. Color-opponent cells are found between the retina and cortex, namely, in the ganglion and lateral geniculate cells (Wiesel and Hubel, 1966; De Valois and Pease, 1971). More complex double-opponent cells having both red-on center and red-off surround combined with green-off center and green-on surround are found in the ganglion cells of goldfish and the cortical cells of primates (Daw, 1968, 1972).

Recent work by Gouras and Padmos (1974) and Gouras (1974) showed that a smaller percentage of cells in monkey cortex exhibits opponent responses compared to the cells in the lateral geniculate. Gouras and Padmos found that graded potentials, the earliest electrical response detectable in the striate cortex of anesthesized rhesus monkeys, showed color-opponent antagonism between cone mechanisms. Gouras (1974) found that many of the cells in the foveal striate cortex exhibited spatial antagonism within the same cone mechanism; if the center of the cell's receptive field was excited by red cones, the surround was inhibited by red cones. There appears to be a decrease in opponent responses at more complex levels.

Whatever the actual properties of the physiological structures and their interactions, the system as a whole works as a reflectance-finding device, as shown in this paper. The information reaching the receptors in the retina is a spatial arrangement of radiances. The sensations reported by observers show little correlation with the wavelength-radiance distributions, but show high correlation with three reflectances measured with light detectors that have the same spectral sensitivities as the three cone receptors. In achromatic situations, lightness is the sensation produced by the reflectance-calculating mechanism of the visual system. Retinex theory proposes that color sensations are dependent on three lightnesses calculated from the wavelength-radiance distributions on the retina. The model described in this paper provides a mathematical description of a process whose major assumptions are that lightness can be calculated by intercomparison of information from a single region of the spectrum, and that color sensations are generated by subsequent comparisons of three lightnesses. The model's predictions agree with the observers' sensations.

Acknowledgements—We are indebted to Edwin H. Land for his many discussions and suggestions throughout these experiments. We are also indebted to R. Clark Jones for his initial suggestion of scaling reflectance so that it is equally spaced in terms of subjective lightness and to Edward M. Purcell for his initial computer design of the model for lightness. We wish to thank Marie A. Watson and Julius J. Scarpetti for their help in preparation of this manuscript.

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